

# **Hong Kong Property Cycle – a frequency domain analysis**

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## **Abstract**

Property cycle study is a popular topic in the current real estate literature, particularly when the market is near the peak or in low ebb. In this study, some of the stylized facts of the Hong Kong property cycle will be examined and high frequency (monthly) data, partly public and partly proprietary, will be used.

Spectral analysis, uni-variate and bi-variate, will be employed to investigate individual cycle and co-movements of two different cycles respectively. Aperiodic movements of price, rental and total returns for various segments of the real estate market of Hong Kong are found. This result could have important implication for the investors who are thinking of investing in real estate: on an investment horizon of several years, they can buy near the trough and sell near the peak.

Co-movements of the direct (various segments) and indirect real estate are also investigated and it is found that residential market is the one that carries the greatest coherence with the indirect market. Retail and office market follow in the pack and the industrial market is least coherent one.

Key words: Hong Kong property cycle, spectral analysis, co-movement

## **Introduction**

The study of property cycle has cropped up from time to time and the pressure for its study is most intense when the property market is in low ebb. When the market is in the upswing, most market participant(s) would wish the market to continue its current course and any idea that the trend may be reversed would be against the wishes of the crowd. The hope that a cycle exists is strongest when the market is at or near its lowest, normally when the market sentiment is of utmost pessimism. In the western world, the outcry for the study of property market was loudest after the crashes of property market in the early seventies and the early nineties. The importance of the study of property cycle can obviously not be underestimated. It has direct linkage to the well being of both individual and institutional investors. If an investor can have a reasonably good feel of the market trend, he/she can then practice the obvious

investment strategy of 'buying low and selling high'. On the defense side, the understanding and awareness of the existence of property cycle can certainly alert one from imminent property crash which happened from time to time in property history. The RICS report on "Understanding the Property Cycle" (1994) was dedicated "to all those who saw their jobs destroyed, or their livelihoods blighted, by the property crash of the early 1990s" clearly demonstrated that is the case. From the perspective of government policy, the intention of reducing the degree of swing (volatility) of the property market is certainly great especially when the housing market is at stake as that would affect the support received by government. Pattie (1995) has found that "...the failure of the housing market was a factor in reducing support for the government. Negative equity is identified as a particularly important problem in this regard." Similar situation happened in Hong Kong after the Asian Financial Crisis and Tung's administration was and is still being accused of causing the widespread negative equity housing problem in Hong Kong as a result of his notorious 85,000 housing units per annum policy.

## **I. Literature Review**

The study of property cycles has a long history. The first known study was made by Hoyt (1933), on the real estate market of Chicago, U.S. for the one hundred years prior to 1933. In the United Kingdom, the study on property cycle went back to 1921. In RICS research report (1999), it used "a new history of property returns stretching back to 1921 to examine key features of property performance: its cyclical characteristics, links between property and economic cycles, and comparisons with the returns produced by other assets ..."

Let us start off the review process with definition of property cycle. Very few current literature pay attention to this, presumably most property cycle researchers take this matter trivial and consider an intuitive definition will do. In RICS report on "Understanding the Property Cycle" (1994), it draws analogy with the study of business cycle and offers the following similar but slightly modified definition: "Property cycles are recurrent but irregular fluctuations in the rate of all-property total return, which are also apparent in many other indicators of property activity, but with varying leads and lags against the all-property cycle." This is by far the most concise definition proposed. It is noted that in this definition it is the 'total return' derived from property that counts.

Previous studies were primarily concerned with answering a number of research

questions. First, is there a property cycle for certain segment of real estate in certain country or place? Secondly, if property cycle exists, what are the causes for its existence? Thirdly, what are the phases of and accompanying characteristics of property cycle? Fourthly, what are the relationships of property cycle observed with business cycle or other economic cycles, i.e. the co-cycle study? Fifthly, can one construct early warning indicator(s) which could correctly identify the turning points of the property cycle and hence lead to better investment decision? Sixthly, can one develop a trading strategy by making use of the 'identified' early warning indicator(s) to exploit excess return from the real estate market? Seventhly, would the market conditions of one place or property type transmitted to other places, both regionally or nationally, or to other property type? A review of the current literature revealed that most of the researchers attempted to answer the above questions in an empirical way with the assistance of contemporary econometric methods. Theorizing the empirical findings seems still in an early stage.

A number of researchers work in the area of 'stylized facts' of the property cycle and co-cycles studies. Wilson and Okunev (1999) used "conventional spectral analysis techniques to examine property and financial assets for evidence of cycles and co-cycles", wherein securitized property index (price) series data from USA, the UK and Australia were employed. Although the study lent support to the existence of cyclical patterns in individual series "... the variance at the identified cycle length was not large, generally of the order of 10 per cent of series variances ... One implication of this is that the very pronounced cyclical patterns that appear in direct real estate markets and the economy as a whole are very much less obvious once they have filtered through to securitized property markets and financial assets markets." The correlation of the stock market indexes and the respective securitized property index series are then studied by co-spectral analysis. Although "the co-spectral analysis for the various financial assets series offered some support for the existence of co-cycles, the co-variance of the price series was again of the order of 9 to 10 per cent of the overall series co-variance.

Brown and Kim (2001) made use of the same analytical technique on Singapore price series data. Their study indicated that the prices for the commercial real estate and property stock exhibit cyclical patterns respectively. Their study also looked at "the cyclical interactions between commercial real estate and property stock market...", i.e. they were interested in the relationship between direct (un-securitized) and indirect (securitized) real estate market rather than the correlation between the economy and the real estate market at large as Wilson and Okunev did. They found that "the

individual spectra indicate that the prices for the commercial real estate and property stock exhibit cyclical patterns ...Evidence from the coherency and cross-amplitude spectra suggests significant price co-movement between the two markets in the long run...”

Wang (2003) ‘examines cycles and common cycles in property and related sectors in the frequency domain’ and employs spectral analysis as its tool too. ‘The findings indicate that property shares common cycles with a number of economic sectors and, in particular, with those sectors that are the user markets of property...The property market swings more severely than the economy as a whole. However, fluctuations in the property market are considered moderate relative to those in the housing market.’

In this study, we will look at some of the stylized facts related to the various segments of the Hong Kong property market.

## **II. Methodology**

The development of time series analysis comes from two directions. Statisticians develop from the side of making statistical inferences based upon the least squares or maximum likelihood theory, the ‘correlation’ or time domain approach. Communication engineers, on the other hand, try to look at the frequency composition of ‘signal(s)’ and are concerned with the ‘spectra’ of the signal(s) or frequency domain approach.

For time domain analysis, we normally have to handle stochastic process and an important class of stochastic processes is stationary process. A time series is said to be strictly stationary if the joint distribution of  $X(t_1), \dots, X(t_n)$  is the same as the joint distribution of  $X(t_1 + \tau), \dots, X(t_n + \tau)$  for all  $t_1, t_2, \dots, t_n, \tau$ . In other words, the joint distribution depends only on the intervals between  $t_1, t_2, \dots, t_n$ . The said definition holds for all  $n$  values. If  $n = 2$ , the auto-covariance function  $\gamma(t_1, t_2)$  depends only on  $t_2 - t_1$  and can be written as  $\gamma(\tau)$  and  $\gamma(\tau) = \text{Cov}[X(t), X(t + \tau)]$  is called the auto-covariance coefficient at lag  $\tau$ . If we normalize the auto-covariance function by  $\gamma(0)$ , we have the so called auto-correlation function (ACF)  $\rho(\tau) = \gamma(\tau) / \gamma(0)$  which measures the correlation between  $X(t)$  and  $X(t + \tau)$ . The corresponding sample terms for auto-covariance function and auto-correlation function are called auto-covariance coefficient ( $c_k$ ) and auto-correlation coefficient ( $r_k$ ) respectively. The plot of auto-correlation coefficients ( $r_k$ ) against  $k$  (lag) is a graph called correlogram which is

useful for making a first guess on the appropriate model for the underlying process. This is because different known models such as AR, MA, ARMA or ARIMA exhibit distinct pattern for ACF. At times, PACF (partial auto-correlation function) may have to be employed when sharper discrimination of models has to be made.  $\rho_{kk}$  measures correlation between observations that are  $k$  periods apart after controlling for correlations at intermediate lags, i.e. lags less than  $k$ . Inference made based upon the interpreting the correlogram, with the assistance of ACF and PACF, so as to identify what type of ARIMA model gives the best representation of the an observed time series, is normally called time domain analysis for the case of single equation modeling. For the case of single equation modeling, Box-Jenkins procedure is normally followed to make forecasts. For the case of simultaneous equation modeling in the time domain analysis, we have the VAR (vector auto-regression) method instead.

Spectral analysis may sound veer initially, but it can actually relate to daily life. When we look at some mono-chromatic (single color) light source, we will feel the ‘strength’ and the ‘color’ of it. The color of the light source reflects the frequency of the light emitting from the source. In this instance, our eyes behave as crude ‘spectrometer’, namely, a meter for measuring ‘spectra’. In simple terms, it can differentiate different ‘color’ of light. When the light source emitting white light, our eyes, due to its crudeness, cannot see the different ‘colors’ contained in it. We need some other better tools to help use the see the frequency composition of it. We need a prism! Spectral analysis plays the role of a prism when we try to look at the frequency composition of a time series.

Analysis of a time series would naturally require the decomposition of the same into trend, seasonal, cyclical and residual components. The first obvious way of analyzing time series data is to look at the plotted diagram of the same. For an experienced eye, it may see whether an obvious trend exists or not. If it does, the data has to be de-trended first before further analysis. There are a number of de-trending techniques; the common ones are first differencing, second-order (or even third-order) polynomial regression or Hodrick-Prescott filtering. It should be noted that the de-trending procedure is equivalent to putting the data through a (linear) filter which may affect the frequency characteristic of the original time series. We have both low band and high band filters which have the property of allowing low frequency and high frequency component to pass through more easily. The residual series after de-trending would be the one that has subject to spectral analysis, testing for the existence of cycles.

General outline of both analysis are as follows. For uni-variate spectral analysis, we assume that the realization of the underlying data generating process, which may be deterministic or stochastic in nature, can be adequately represented as a sum of sinusoidal oscillation of different amplitude and related frequencies, i.e. the realization is a combination of sinusoidal functions of different but somehow related frequencies. The analysis of the data of the underlying generating process by frequency is said to be analysis in the frequency domain. This is in contrast to the study of auto-covariance function etc. of the original time series in the time domain.

**Uni-variate analysis**

By Wiener-Khintchine theorem, any stationary stochastic process with autocovariance function  $\gamma(k)$ , there exists a monotonically increasing function  $F(\omega)$  such that

$$\gamma(k) = \int_0^\pi \cos \omega k \, dF(\omega) \tag{2}$$

Equation (2) is called the spectral representation of the auto-covariance function.  $F(\omega)$  has an important physical interpretation: it is the contribution to the variance of the series which is accounted for by frequencies in the range  $(0, \omega)$ .

If we use  $f(\omega)$  (assuming it exists) to denote the derivative of  $F(\omega)$ , i.e.

$$f(\omega) = \frac{dF(\omega)}{d\omega}$$

then we have come up with a very important function for spectral analysis. It is the (power) spectral density function or shortened to ‘spectrum’. Equation (2) can then be rewritten as:

$$\gamma(k) = \int_0^\pi \cos \omega k \, f(\omega) \, d\omega \tag{3}$$

The physical meaning of the spectrum is that  $f(\omega) \, d\omega$  represents that contribution to variance of components with frequencies in the interval  $(\omega, \omega+d\omega)$ . Equation (3) expresses  $\gamma(k)$  in terms of  $f(\omega)$  as a cosine transform. The inverse relationship is given by

$$f(\omega) = \frac{1}{\pi} \int_{k=-\infty}^{\infty} \gamma(k) \, e^{-i\omega k} \, dk$$

The spectrum is thus the **Fourier transform** (see Appendix 1) of the auto-covariance function. Spectrum is normally estimated by periodogram, the definition of which is

at appendix 2. However, the periodogram is deficient in the fact that it is not a consistent estimator of spectral density function although it is asymptotically unbiased. This means that as  $N$  goes to infinity, the variance of  $I(\omega)$  does not go to zero.

In order to have a consistent estimate of the (power) spectral density function, one has to go through some sort of smoothing procedure. It is noted that the periodogram is the discrete Fourier transform of the complete sample auto-covariance function. An estimate of the following form would normally be used.

$$\hat{f}(\omega) = \frac{1}{2\pi} \left\{ \lambda_0 c_0 + 2 \sum_{k=1}^M \lambda_k c_k \cos \omega k \right\}$$

where  $\{\lambda_k\}$  are a set of weights called the lag window and  $M$  is called the truncation point. Different types of window are developed such as Tukey, Parzen, and Hanning etc.

### **Bi-variate process**

When we want to look at the relationship between two time series, we have bivariate processes to deal with. There are two different types of situation for it. In the first situation, the two series arise on the equal footing with the possibility that both arise from the same underlying disturbances. On the other hand, one of the series is regarded as the input whereas the remaining one the output of a linear system, in the case of second situation. The first type can be said as the equivalent of correlation whereas the second type that of regression.

Suppose we have  $N$  observations for two different series  $\{x_t\}$  and  $\{y_t\}$  at unit time intervals over the same period. The observations may be denoted by  $(x_1, y_1), \dots, (x_N, y_N)$ . These observations may then regard as a realization of a discrete bivariate process  $(X_t, Y_t)$ . Similar to the univariate case, we have up to second order moments i.e. mean and auto-covariance function for each of the two components. In addition, we have a new function called the cross-covariance function, given by:

$$\gamma_{xy}(t, k) = \text{Cov}(X_t, Y_{t+k})$$

The complementary function in the frequency domain is called **cross spectral density function** or **cross-spectrum**. Similar to the uni-variate case, the cross-spectrum of a discrete bivariate process measured at unit intervals of time as the Fourier transform of the cross-covariance function is defined as:

$$f_{xy}(\omega) = 1/2\pi[\Sigma\gamma_{xy}(k) e^{-i\omega k}]$$

over the range of  $(0, \pi)$ . It is observed that  $f_{xy}(\omega)$  is a complex function and the inverse relationship is

$$\gamma_{xy}(k) = \int_{-\pi}^{\pi} e^{-i\omega k} f_{xy}(\omega) d\omega$$

To give a proper interpretation of the cross-spectrum, let us look at both the real and imaginary part of it. The real part of the cross-spectrum is called the **co-spectrum** and is given by:

$$c(\omega) = 1/\pi[\Sigma\gamma_{xy}(k) \cos\omega k]$$

The imaginary part of the cross-spectrum, with a minus sign, is called the **quadrature spectrum** and is given by:

$$q(\omega) = 1/\pi[\Sigma\gamma_{xy}(k) \sin\omega k]$$

$$\text{and } f_{xy}(\omega) = c(\omega) - i q(\omega)$$

Another function derived from the cross-spectrum is the (squared) **coherency**, which is given by:

$$\begin{aligned} C(\omega) &= [c^2(\omega) + q^2(\omega)]/[f_x(\omega) f_y(\omega)] \\ &= \alpha_{xy}^2(\omega)/ f_x(\omega) f_y(\omega) \end{aligned}$$

where  $f_x(\omega)$ ,  $f_y(\omega)$  are the power spectra of the individual processes. It can be shown that  $0 \leq C(\omega) \leq 1$  and it measures that square of the linear correlation between the two components of the bivariate process at frequency  $\omega$ . It is therefore analogous to the square of the usual correlation coefficient. This property makes it suitable for the measurement of the co-movement of the components of the bi-variate process.

### **III. Data**

Hong Kong property market is enriched by a wealth of data. In this study, we will use the monthly data (since January 93) published by the Rating and Valuation department (RVD) of Hong Kong SAR government. As the RVD published only quarterly data prior to January 1993, monthly data prior to the said date was obtained by interpolation. The database of RVD covers both price index, rental index and yield level. By combining the price index and rental index provided by the RVD, we manage to construct a total return index series for various real estate market segments.

We have another set of monthly data on price obtained by repeated sales method based on the seminal paper of Bailey (1985). This set of data is proprietary in nature and was constructed by extracting actual transaction details from land registry since the beginning of 1970s. Due the relatively low volume of transactions, the volatility of the variance of the price data was greater in the early 1970 and became smaller when number of transactions collected for calculation was larger at a later time.

The natural logarithms of the price series were first subject to Hodrick-Prescott filtering in order to remove the trend element before they were put up for testing of the existence of cyclical elements by spectral analysis. All the series: price, rental and total return series, were tested for their respective stationarity at 10% significance level. As a matter of fact, some of them even met 1% significance test.

## **IV. Results**

### **5.1 Single cycle movement**

By looking at the price indexes (and their smoothed trends) of the various market segments of the property market of Hong Kong at Figure 1, we observed that they do undergo upswing and downswing period. This provides *prima facie* evidence for the existence of cyclical phenomena for the property market.

In order to check the cyclicity of the price movements, we apply the HP filter to de-trend the time series and then apply spectral analysis to the price series to obtain the relevant spectra. The spectral diagrams in Figure 2 leads us to believe that there are indeed cycles for the various segments: residential, retail and office are of 36 months, 36 months and 50 months respectively. However, there is no clear identifiable cycle for the industrial segment. It is speculated that this may due to the continued declining demand for industrial properties in Hong Kong as most of, if not all, industrial undertakings move to the mainland.

We also looked at the cyclical movements of rentals of various market segments. The rental data we have are current ones and not those reserved on the lease and hence provided good indication on rental market conditions at different times. The rental market is different from the sale market (investment market) in the sense that it predominately reflects the occupier market conditions. The rental markets for the various market segments do exhibit cycles of approximately the same lengths of 4 years as shown in Figure 3.

The total returns series of the various market segments, checked to be stationary series, were also subject to the spectral analysis. The resultant periodograms were in Figure 4. It is noted that for all market segments, some kind of 4 years cycle was observed.

### **5.2 Co-cycle movements**

As regards to the co-movement of direct and indirect real estate, we are primarily concerned with the price movement of direct real estate, as measured by the price indexes of the various market segments, and indirect real estate, as proxy by Hang Sang Property Indices (HSPI).

Co-movement tests were conducted to see the correlation relationship between the various real estate market segments and HSPI and the results are shown in Figure 5.

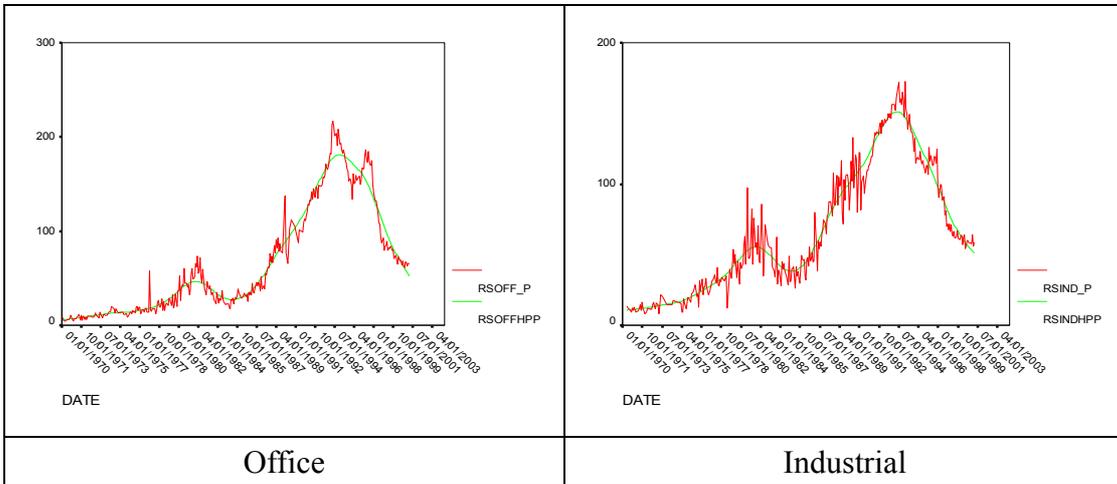
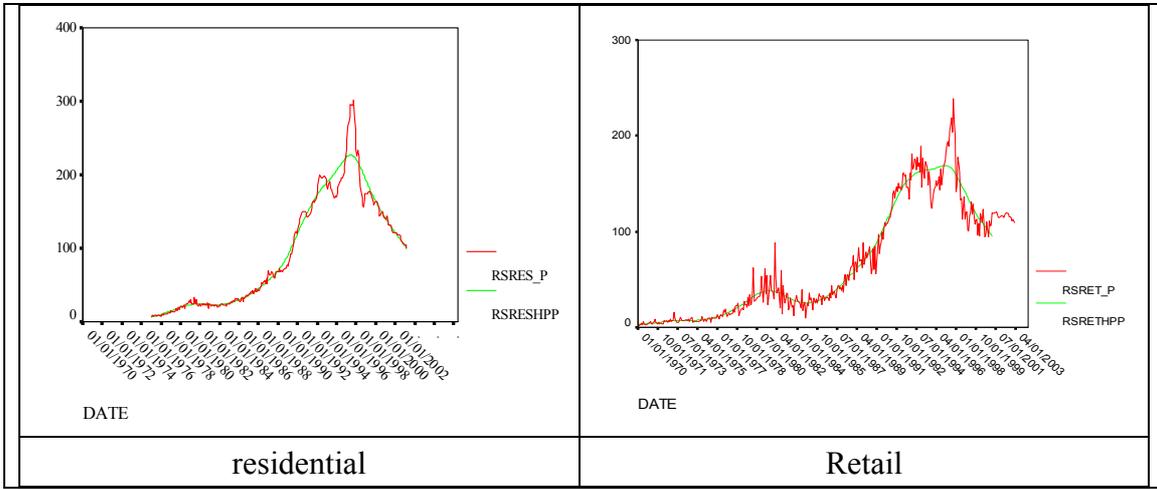
It is observed that the residential real estate segment has the greatest coherence value with the HSPI, followed by the retail and office segment and industrial is the least coherent one. It is speculated that this may be due to the relative weightings of investments / developments placed by the property companies forming the HSPI on the different market segments.

## **V. Conclusions and future research directions**

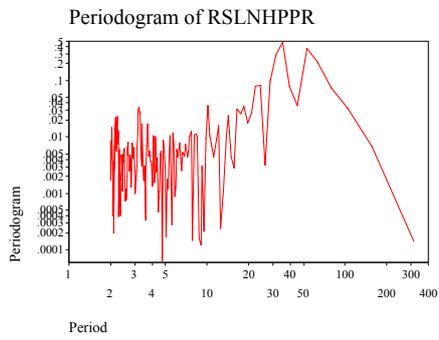
In our study, we found that cycles do exist in the various real estate market segments in Hong Kong. This observation will provide opportunity for real estate investors, with investment horizons of several years, to reap excessive profit. They could have the opportunity of buying near the bottom of and sell close to the peak of the market. This is somewhat in contradiction with the Efficient Market Hypothesis. However, recent finance literature revealed that EMH tested on short term data and long run data would provide different results and it is possible that even the stock market would exhibit inefficiency on the long run data of say 5 years. It is therefore worthwhile to borrow latest research findings of equity market, both theoretical and experimental, to the real estate market.

Our study covers also the co-movement of the direct real estate, as measured by the price indexes of the various real estate market segments, with the indirect real estate, proxy by the HSPI. We found that residential market segment bears the greatest coherence with the indirect investment, followed by the retail and office market segments, and with the industrial comes last.

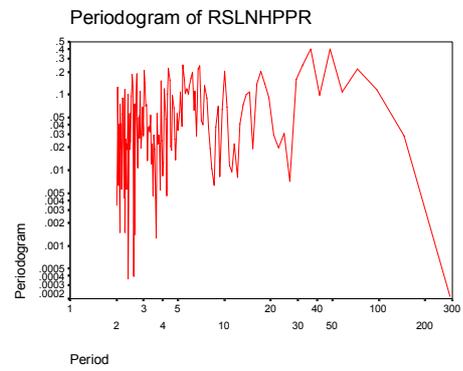
**FIGURE 1**



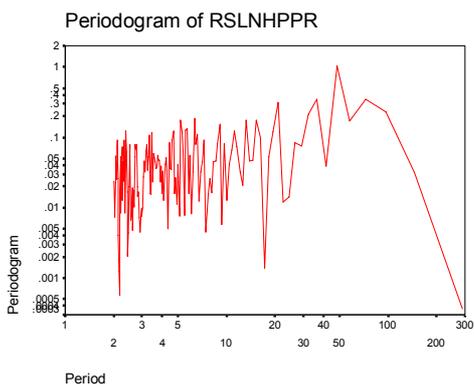
**FIGURE 2**



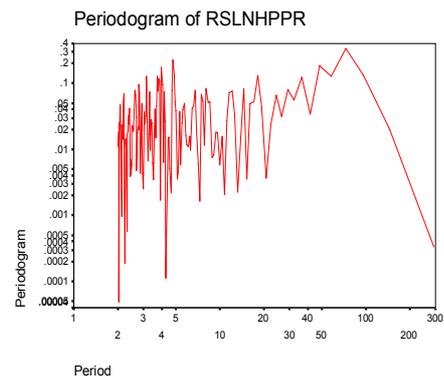
Residential



Retail

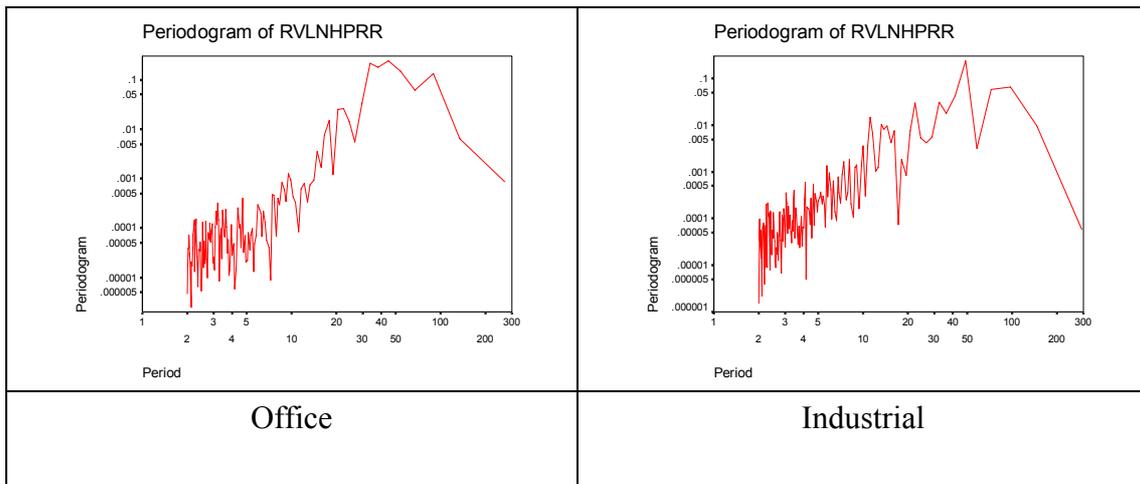
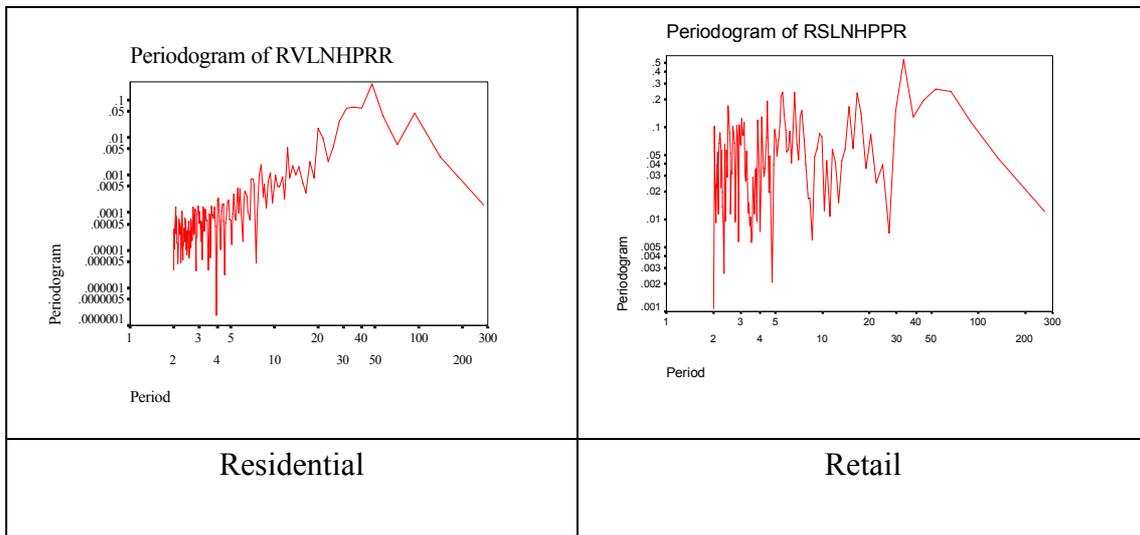


Office

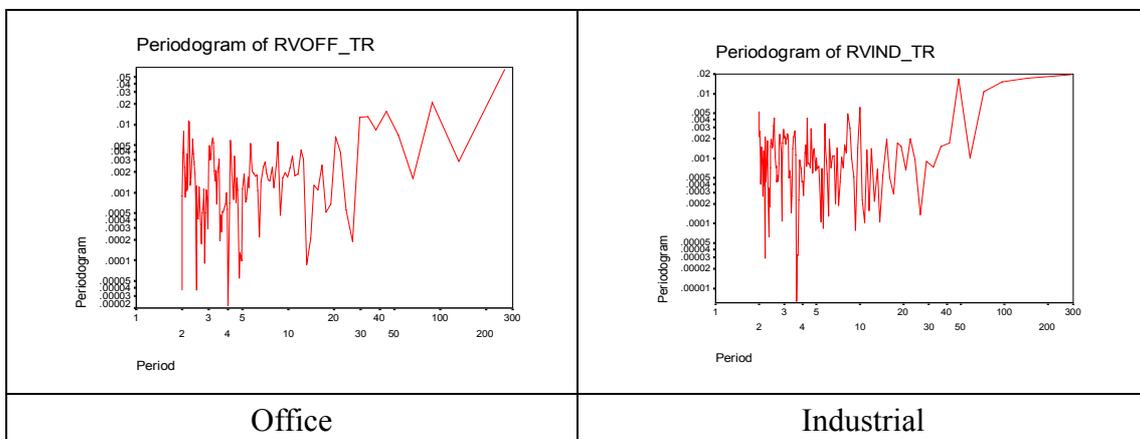
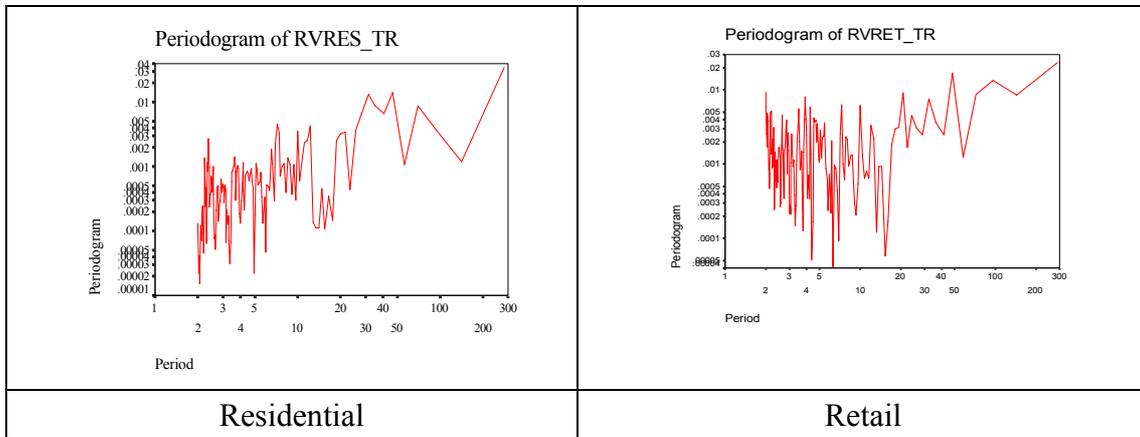


Industrial

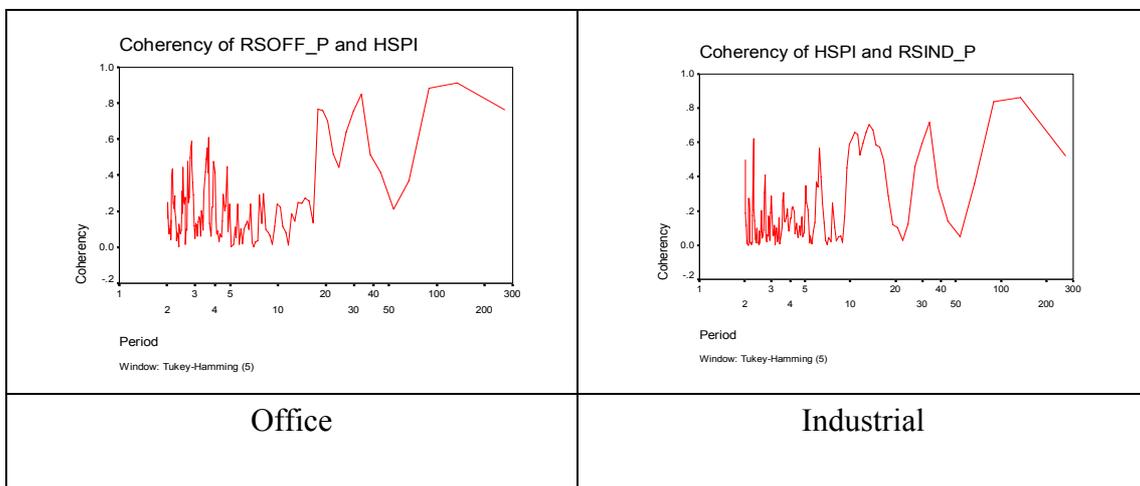
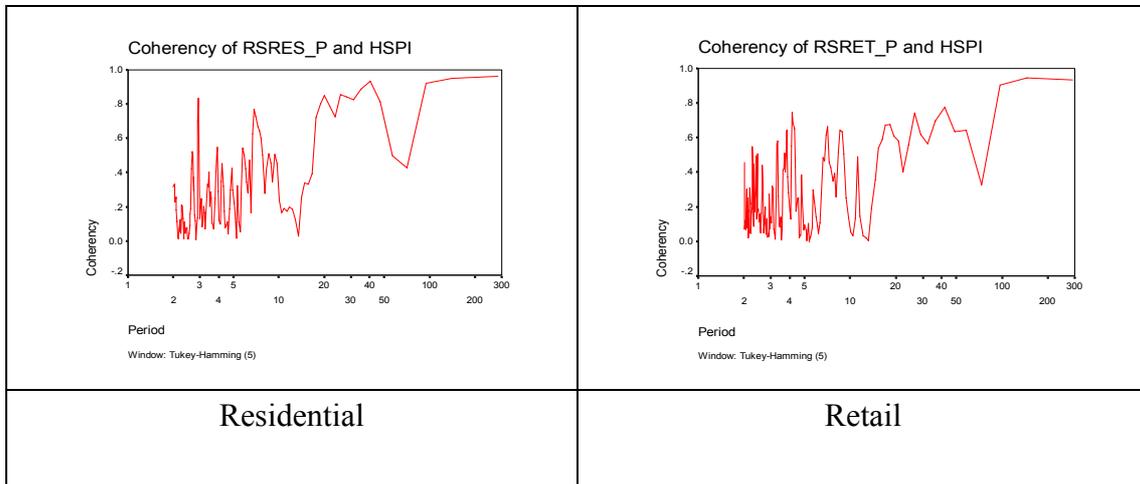
**FIGURE 3**



**FIGURE 4 ( Total Returns)**



**FIGURE 5**



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## Appendix

### FOURIER TRANSFORM

#### Continuous Fourier Transform

Given a function  $h(t)$  of a real variable  $t$ , the Fourier transform of  $h(t)$  can be defined as

$$H(\omega) = \int_{-\infty}^{+\infty} h(t) e^{-i\omega t} dt \quad (1)$$

provided the integral exists for every real  $\omega$ .

A sufficient condition for  $H(\omega)$  to exist is

$$\int_{-\infty}^{+\infty} |h(t)| dt < \infty$$

If (1) is regarded as an integral equation for  $h(t)$  given  $H(\omega)$ , then a simple inversion formula exists of the form

$$h(t) = 1/2\pi \int_{-\infty}^{+\infty} H(\omega) e^{i\omega t} d\omega \quad (2)$$

and  $h(t)$  is called the Fourier transform of  $H(\omega)$ .

#### Discrete Fourier Transform

In time series, we commonly use the discrete form of the Fourier transform when  $h(t)$  is only defined for integer values of  $t$ . Then we have:

$$H(\omega) = \sum_{t=-\infty}^{\infty} h(t) e^{-i\omega t} \quad -\pi \leq \omega \leq \pi$$

is the Fourier transform of  $h(t)$ . Note that  $H(\omega)$  is defined only in the interval  $[-\pi, \pi]$ .

The inverse Fourier transform is given by:

$$h(t) = 1/2\pi \int_{-\pi}^{\pi} H(\omega) e^{i\omega t} d\omega$$