

MULTIPLE CRITERIA DECISIONS: OPENING THE BLACK BOX

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Abstract

Multiple criteria decision tools are now widely used thanks to the availability of good commercial software packages. However, from a pedagogical point of view, such instruments are complete black boxes that would require a level of mathematics far beyond the reach of most users.

This paper, of a strictly pedagogical nature, suggests that MCD can be taught and tested simply by using our good old Excel spreadsheets. The paper also suggests an innovative and Excel based method to measure the coherence of multiple criteria matrices.

Keywords: Multiple criteria decision, Analytical Hierarchy Process (AHP), Determinants, Teaching and learning tools, Office location, Kuala Lumpur

1. Introduction

The paper's objective is strictly pedagogical and it was prepared for the benefits of those of us who have to convince students of the 'usability' of MCD instruments.

Typically, when we teach MCD tools, we explain how it works (more or less) and we conveniently occult the nasty mathematical background. In truth, the mathematical underpinning of MCD techniques is beyond the reach of most run-of-the mill property or even economics students.

When budget and time allows, we may be fortunate enough to demonstrate the use of some software packages and impress our students with the flexibility and power of MCD. However, these packages are completely 'black boxy' and - if they may impress the users - they certainly do not contribute to the fundamental understanding of MCD tools.

Such tools can, however, be demonstrated to and experimented by students who have some basic Excel skills. This paper will go through the simple calculations required and will even suggest how decision transitivity can be tested without complicated consistency rules *a la* Saaty.

2. Multiple criteria decision: some historical background

Multi-Criteria Decision Analysis (MCDA) is based on computational instruments used to support the subjective evaluation of a finite number of options under a finite number of performance criteria.

Decision makers must judge the options performance under selected criteria and simultaneously they must weight the relative importance of the criteria in order to arrive at a global judgment. The archetypal example used in MCDA being the purchase of a car. How can a purchaser choose between unrelated criteria such as price, power, colour, gadgets, trunk volume and subliminal appeal?

The MCDA technique has been used of in operation research for at least 30 years starting with the pioneering work of (Bell, Keeney et al. 1977), (Saaty 1980), (Roy 1985), (von Winterfeldt and Edwards 1986).

Over the years, the various emerging approaches have diverged. Four perspectives can now be identified.

- A descriptive perspective that tells us how decisions are made by decision makers faced with complex set of options under conflicting viewpoints;
- A normative perspective that suggests how decision makers should behave and how MCDA should work, via logical rules, which are based upon certain fundamental axioms such as the transitivity of preferences. The typical products of the approach are Multi-Attribute Value Theory (MAVT) and Multi-Attribute Utility Theory (MAUT), to be used for decision problems with certain and with uncertain outcomes respectively. This perspective is usually described as the American branch of MCDA
- A prescriptive approach that describes how decision makers could improve the decision process and the decisions themselves, and how MCDA could support such a process.
- A constructive perspective that questions the existence of a coherent, well-ordered system of preferences and values in the decision maker's mind. The constructivists suggest that the decision maker's preferences and values, initially unstable, will be shaped by MCDA. This constructive perspective seems to be favoured by European analysts (Roy 1985). It is also – broadly - the perspective taken by Saaty in the development of the Analytical Hierarchal Process described in this paper.

Initially, MCDA techniques have received very little attention from Urban and Real Estate researchers. As early as 1982, a working paper from this author used MCDA to analyse the decision to purchase office buildings by institutional investors.(Fischer and Moffet 1982).This paper - published in French and using APL - not a very popular programming language - has had no significant impact.

Since then the techniques – aided by the development of friendly computer packages – seem to elicit more enthusiasm. Notably MCDA has be used to explain housing choices (Ball and Srinivasan 1994), to model environmental quality: (Bender, Din et al. 1999)), to determine preferences on office quality attributes (Ho 1999) or to asset stigma effects on land and property values (Chan 2002). In the Pacific Rim sphere, MCD is receiving an increasing exposure as illustrated by (Chan 2002), (Fischer 2003), (Newell and Seabrook 2006) and (Rowland 2007).

As mentioned in the introduction, MCD models - including AHP techniques - rely on quite challenging mathematical basis, however, the general principles are not that mysterious as we plan to illustrate now. Along the way, some basic Excel tricks or manipulations will be indicated since, as we all know, our students do require the teaching and re-teaching of most Excel functionalities.

3. AHP: the easy way.

A simple (and fully contrived problem) is illustrated with the help of a simple Excel worksheet. The decision to be discussed is the choice of an office location between three possible urban

sectors in Kuala Lumpur (CBD, Golden Triangle and Bangsar). The three decision criteria are rent level, situs characteristics and image (area prestige).

Situs characteristics are a mix of accessibility and networking options that – in a fuller analysis – could be broken down in many different variables such as: distance to public transport, distance to eating establishments, distance to complementary businesses, distance to Mosque or Gyms, level of amenities, etc.

A preliminary table should be established and, preferably, the variables should be named (Insert > Name > Create> Left column).

Table 1: Location decisions in Kuala Lumpur

Number of criteria (Rental, Situs, Image)	crit	3
Number of alternative locations (CBD, Golden Triangle (GT) and Bangsar)	loc	3
The 5 level preference scale		
No preference		1
Slight preference		2
Some preference		3
Significant preference		4
Very strong preference		5

On the basis of this preference scaling, the first matrix establish the relative importance of the three criteria.

Table 2: Ranking the criteria

	Rental	Situs	Image
Rental level	1.0000	2.0000	3.0000
Situs advantage	0.5000	1.0000	4.0000
Image	0.3333	0.2500	1.0000
	1.8333	3.2500	8.0000

The first line in table 1, indicates the decision maker preferences. Rent (lower rent) is a slightly preferred criteria (score 2) with respect to situs and somewhat preferred with respect to Image (score 3). Of course, students can choose a completely different ranking and scoring system. Indeed, when the whole sheet is organised, they should be encouraged to modify their initial rankings.

From the second line (last column) we see that Situs is significantly more important (score 4) than Image. The first cell of line 2 is the inverse of the Rent/Situs comparison: it means that if Rental is preferred by a score of 2 over situs. Then, Situs is 'not preferred' by a score of $\frac{1}{2}$ over Rent. The rest of the table is calculated in the same way, noting of course that the diagonal is equal to one since it compares the same criteria.

Now, if you sum the respective columns, the lowest score (here 1.833) indicates the most important criteria in the eyes of the decision maker. Clearly, this decision maker is mostly 'Ringgit minded' and he values lower rent over the other criteria.

The colouring (grey) of the smallest cell is obtained with the 'conditional formatting' functionality. This is a nice gimmick that does not fail to amuse the students.

The next step requires the normalising of the scores. The resulting table is presented now. The calculations are explained later. To facilitate the explanation, the tables are now including their grid numbers and their 'auditing tracks'

Table 3: Weighting the criteria¹

	A	B	C	D	E	F	G	H
1	Ranking the criteria							
2		Rental	Situs	Image	Relative values			Ranking of weighed preferences
3	Rental level	1.0000	2.0000	3.0000	0.5455	0.6154	0.3750	0.5119
4	Situs advantage	0.5000	1.0000	4.0000	0.2727	0.3077	0.5000	0.3601
5	Image	0.3333	0.2500	1.0000	0.1010	0.0769	0.1250	0.1279
6		1.8333	3.2500	8.0000	1.0000	1.0000	1.0000	1.0000

Each criterion is first normalised by dividing each cell (such as B3) but the average for the criteria (such as B6). Then the averages are calculated as in H3:H5. This average represents a preference scaling based on the relative weights of each criterion. In this case we could say that Rental represents 51.19% of the decision maker preferences. The rental level consideration will weight more heavily in the decision. The least important criteria appears to be the image factor (12.79%). Now that we have established the relative weight of each of the three criteria, we can assess the relative evaluation of each location with respect to each criterion.

The next three tables will illustrate the calculations and results. The process is very similar than the one we applied to the criteria relative comparisons. If we were conducting a real research, the pair wise comparison should be obtained from the structured interviews with decision makers. Some coaxing and requestioning may be necessary to assure some coherence in the interviewee's answers. In this example, we assume a fairly high degree of coherence (this issue will be discussed below).

Table 4: Pair wise ranking of the preferences for lower rent and resultant rental ranking for the 3 alternative locations

	A	B	C	D	E	F	G	H
8	Pair wise comparison on Rental level				Weighting the rental level			Rental ranking
9		CBD	GT	Bangsar				
10	CBD	1.0000	5.0000	3.0000	0.6522	0.6250	0.6667	0.6479
11	GT	0.2000	1.0000	0.5000	0.1304	0.1250	0.1111	0.1222
12	Bangsar	0.3333	2.0000	1.0000	0.2174	0.2500	0.2222	0.2299
13		1.5333	8.0000	4.5000	1.0000	1.0000	1.0000	1.0000

¹ A minor note on the use of colours: I am totally allergic to the use of colours in spreadsheets (except grey...). Multicoloured spreadsheets are quite annoying. Colours are used here – exceptionally - to facilitate the visual connections between the tables and vectors. Since, this paper will be printed in black and white, this little pedagogical effect will be lost anyway

Table 5: Pair wise ranking of the preferences for good situs and resultant situs ranking for the 3 alternative locations

	A	B	C	D	E	F	G	H
15	Pair wise comparison of Situs advantage				Weighting the situs advantage			Situs ranking
16		CBD	GT	Bangsar				0.5623 0.3228 0.1149
17	CBD	1.0000	2.0000	4.0000	0.5714	0.6154	0.5000	
18	GT	0.5000	1.0000	3.0000	0.2857	0.3077	0.3750	
19	Bangsar	0.2500	0.2500	1.0000	0.1429	0.0769	0.1250	1.0000
20		1.7500	3.2500	8.0000	1.0000	1.0000	1.0000	1.0000

Table 6: Pair wise ranking of the preferences for a superior image and resultant image ranking for the 3 alternative locations

	A	B	C	D	E	F	G	H
22	Pair wise comparison of Image				Weighting the image factor			Imageranking
23		CBD	GT	Bangsar				0.1179 0.6266 0.2554
24	CBD	1.0000	0.2000	0.5000	0.1250	0.1176	0.1111	
25	GT	5.0000	1.0000	3.0000	0.6250	0.5882	0.6667	
26	Bangsar	2.0000	0.5000	1.0000	0.2500	0.2941	0.2222	1.0000
27		8.0000	1.7000	4.5000	1.0000	1.0000	1.0000	1.0000

Then, we multiply the vector of relative weight to the respective rankings in the matrix of the 3 criteria. This calculation could use the MMULT function and ARRAY entry. This is the elegant way to proceed, however the use of ARRAYS can be problematic when you want then to reformat or reorganise your table. Typically also, students will forget to use the SHIFT + ALT + ENTER before calculating the array.

Here the array function `{=MMULT(B30:D32,E30:E32)}` has been used. However, it may be a better idea to remind students how to multiply a vector by a matrix: they probably have not done this for quite a long time.

Table 7: Multiplying the vector of criteria weights by the 3 vectors of rankings (matrix A29: C32)

	A	B	C	D	E	F
29	Locations	Rental	Situs	Image	Criteria weights	Final ranking
30	CBD	0.6479	0.5623	0.1179	0.5119	0.549
31	GT	0.1222	0.3228	0.6266	0.3601	0.259
32	Bangsar	0.2299	0.1149	0.2554	0.1279	0.192

From column F (Final Ranking) we can now conclude that the multicriteria decision will be the choice of CBD as the best location (wit a 54.9% preference). This outcome was quite predictable since our decision maker was mostly interested in a lower rent level.

Once the tables are created, the students should simulate different preferences and scaling. If the case calculated here was the decision of Mr. Kikir (Skinflint), we could simulate the decision of

Ms. Sombong (Image conscious) and Dr.Malas (situs conscious) by inverting the various criteria weighting and ranking. The other benefits of experimenting with different rankings is that the students will soon be confronted to the problem of transitivity.

4. Measuring transitivity: a flashback to linear algebra.

Can deciders be coherent in their multicriteria decisions? Or to put it more formally, how can we verify if their preferences are transitive². In the very simple example presented here, coherence was not a problem since the matrix was intentionally set up to be almost fully transitive.

However, interviews of real decision makers may lead to much more ambivalent ranking especially when the number of choices and criteria are increased. In fact, total transitivity is impossible to reach in more complex realistic problems.

The Multi-criteria software packages have their own techniques to examine the incoherencies and they may send some 'flag raising' signals to the users when the responses do not make much sense. Typically, the coherence testing is not explained and – if it were explained – it may be based on mathematical criteria that would not be obvious to our typical students.

Here, I will suggest a simple technique that could give the user some control on his evaluation of the coherence of his interviewee's answers. The idea is to compare the determinants of resulting matrixes obtained from the questionnaires with the determinants of matrices that would be obtained by purely random answers to the weighting and ranking questions³.

We start with with a perfectly coherent scoring. We must use the full range of possible answers (from 1 to 5) since we will bracket our random scores also between 1 and 5⁴:

Table 8: Too good to be true: perfect coherence

	Rental	Situs	Image
Rental level	1.0000	3.0000	5.0000
Situs advantage	0.3333	1.0000	0.6000
Image	0.2000	0.6000	1.0000
	1.5333	4.6000	6.6000

Now we establish a completely random decision matrix by entering RANDBETWEEN (1,5) in all the non diagonal cells.

² A decision is transitive if the order of preference is maintained. For example the preference of A to B and of B over C lead to the preference of A to C.

³ Comparing with random results had been initially suggested by Saaty (1980) in order to calculate consistency indexes. The Saaty coefficients are calculated in appendix 2. This could help the students... or it may not. I suggest that the Determinant method has more intuitive appeal.

⁴ A perfectly coherent matrix can be obtained by taking the ranking of the (North East corner of the matrix) and dividing them pair wise to obtain the South West corner. See the calculation in appendix 1.

We know calculate the determinant of this matrix with the function {MDETERM} (yes, another tricky ARRAY function). The determinant of perfectly coherent matrix is equal to zero (see appendix 1).

By contrast, the user can now generate a number of calculations of 'random' determinants (using F9 to recalculate the random matrix). The determinants of 'random matrices' will be a large positive or negative numbers. Try it....

Table 9: Random incoherence

	Rental	Situs	Image
Rental level	1.0000	1.0000	3.0000
Situs advantage	5.0000	1.0000	5.0000
Image	3.0000	1.0000	1.0000

The determinant of this randomly generated matrix is 12. Using this range (1 to 5) of possible answers in a 3*3 matrix, random determinants can go over a 100 or can become negative down to 60. Thus we clearly have a 'coherence' testing based on the simple calculation of the determinants.

The cutting point between total coherence and total incoherence may be difficult to establish in theory⁵ however - in practice – we have our 'flag-raising' signal that should make us suspicious of our questionnaires coherence if the determinants are much larger than zero. We may even try to prod our interviewees to re-examine their answers if the results are indeed too intransitive.

5. Conclusion

Multiple criteria decision models can be explained using basic Excel skills. Even transitivity rules can be simulated with the help of another – and rarely used – Excel function. The process is fully transparent and under the user's control. This transparency and control contrast with the 'black boxness' of commercial multicriteria software packages. Once the students have a good intuitive understanding of the MCD principles then the software packages could be used more effectively.

Still, some of your inquisitive students may want to know more about determinants and coherence thresholds. Well... you may have to wing it and explain that like with Russian dolls, one always find a small black box tucked inside the big black boxes.



⁵ A high value for the determinant indicates a large asymmetry in the matrix South West corner. However, the degree of 'incoherence' cannot be readily determined. (Or a least is beyond this writer skills in linear algebra...).

Appendix 1: Using determinants to test coherence

The determinant of a three by three matrix such as A

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Is written:

$$= (aei + bfg + cdh) - (gec + hfa + idb)$$

A non - inversible matrix has a determinant equal to zero. In our case, a perfectly transitive matrix is made up of linear combination of the three lines and will have a zero determinant.

Let us illustrate the problem with Excel calculations

If we create a perfectly coherent⁶ criteria matrix A such as:

Table 10: A perfectly coherent criteria matrix

	Rental	Situs	Image
Rental	1	2	3
Situs	0.5000	1.0000	1.5000
Image	0.3333	0.6667	1.0000

From the formula above, the decomposition of the determinant would yield:

$$A = 3 - 3 = 0$$

If, on the other hand we produce a random matrix A* such as:

Table 11: An 'incoherent' matrix

	Rental	Situs	Image
Rental level	1.0000	3.0000	2.0000
Situs advantage	3.0000	1.0000	5.0000
Image	3.0000	3.0000	1.0000

The decomposition of the determinant will now yield:

$$A^* = 64 - 30 = 34$$

⁶ This matrix is 'perfectly coherent' because not only the transitivity is maintained but also the ratios of preferences are maintained. The same results would obtain with the inversion of scales or with non linear scaling.

Thus, any determinant greater than zero diverges from perfect transitivity in the ranking matrix.

However, this method is not foolproof since the repetitive generation of random matrices can occasionally produce a zero determinant (with a 'symmetrical incoherence'). Again, here, you should have the students try it and observe the results. The solution may thus not be totally robust, however, from the next appendix we can also observe that Saaty's solution has its own degree of vagueness and it does require more obscure calculations.

Appendix 2: Saaty consistency ratios

Translated in Excel with the same data used above, the Saaty procedure⁷ requires the following steps:

1. Your first calculate the relative weights for the criteria, and then, for each of the criteria with respect to the three locations. Only the criteria and the rental are illustrated here (the others are calculated in the same fashion).
2. The weight vector is obtained by the multiplication of the ranking by the preference matrix (another array formula to use here). Then the weighted sums are obtained by dividing the weights by the ranking (as in $3.15 = 1.6160/0.5119$). Finally, the average weight is obtained (here: 3.1087).

Table 12: Weighting the criteria

Relative values			Ranking of weighed preferences	weights vector applied to criteria	Weighted Sum
0.5455	0.6154	0.3750	0.5119	1.6160	3.1565
0.2727	0.3077	0.5000	0.3601	1.1278	3.1315
0.1818	0.0769	0.1250	0.1279	0.3886	3.0380
				average weight	3.1087

3. The same steps are taken for – as an example – the Rental level factor:

Table 13: Weighting the Rental level with respect with CBD, GT and Bangsar

Relative values			Rental priority	Weighted priority for rental	Weighted sum
0.6522	0.6250	0.6667	0.6479	1.9485	3.0071
0.1304	0.1250	0.1111	0.1222	0.3667	3.0013
0.2174	0.2500	0.2222	0.2299	0.6902	3.0026
				average weight	3.0037

Table 14: the average weights calculated for all the criteria for the three locations

average weight for criteria	awcrit	3.1087
average weight for rent	awrent	3.0037
average weight for situs	awsit	2.9434
average weight for image	awima	3.1532

4. Then the Saaty's consistency indexes are calculated from the suggested formula⁸

(Average weight – number of criteria)/(number of criteria -1)

⁷ Excel demonstrated in Stevenson (2007).

⁸ Saaty (1980), p. 21

Since we have 3 criteria, the result would be:

Table 15: Coherence indexes from all the matrices

Coherence index for criteria	cicrit	0.0543
Coherence index for rental	cirent	0.0018
Coherence index for situs	cisitus	-0.0283
Coherence index for image	cimage	0.0766

5. Finally, these results are compared to the same index obtained from random matrices and a ratio is calculated as:

Consistency ratio = Coherence index for a given criteria/ Coherence index from a randomly calculated matrix.

Saaty suggests that a ratio greater than 0.10 should raise some suspicion and may require a resubmission of the questions. Thus this solution is a 'flag raising' type of result of the same nature as my determinant method described in appendix 1.

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