Theory for Real Estate Valuation: An Alternative Way to Teach Real Estate Price Estimation Methods

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Abstract

Although people often talk as if theory and practice are different things, as in "that is only theoretical," nothing is more practical than a good theory. Theory helps make sense of complex situations by directing attention to key issues and by guiding methods of analysis. This paper presents an updating of valuation theory and the methodological implications flowing from this theory. The central idea is that instead of teaching based around three approaches to value we should base teaching on concepts of price distributions, pricing models and prediction error analysis. This grounds real estate valuation more firmly in modern economics and finance theory and statistical methods as they have developed in recent academic literature.

Outline of the argument

In an *American Economic Review* paper, Peter Kennedy complained that after their first econometrics course students can often use formulas to get answers, but lack understanding needed for practical applications (Kennedy, 1998). Kennedy uses the term "constructivism," meaning that people construct a version of reality that helps guide their thinking and even perception. The same "facts" can be understood in various ways. He suggests that our constructions of reality act like a "lens" to focus thinking. Most students do not think statistically and never acquire a construction of econometrics that enables them to understand how it works and to interpret the meaning of results.

Kennedy suggests *sampling distribution*[1] as the key construct that can focus thinking correctly. Other key concepts are *probability distributions*[2] *and estimators*[3]. He recommends that teachers of econometrics reallocate time to

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[1]"We need to be able to measure how close the sample mean is likely to be to the population mean. The sampling distribution…plays a key role in statistics, because the measure of proximity it provides is the key to statistical inference." (p. 289) Keller and Warrack, *Statistics 4th Ed.* .

[2] A probability distribution can be represented by a graph with the value of a variable on the x axis—for example a property price—and a probability density function on the y axis. Area under the curve shows probability of a value between any two prices.

[3] Estimators are sample statistics used to estimate values of population parameters such as the mean or standard deviation. Desirable properties of estimators are that they be unbiased and consistent (that is, they approach the population value as sample size increases).

teaching students these key ideas. This paper applies Kennedy's recommendations to property valuation.

The profession of real estate valuers arises because each real estate asset is different from all other properties. Real estate assets are heterogeneous, that is, their characteristics vary. Researchers and practitioners have found that hundreds of factors might affect prices in various situations.[4] Moreover, properties trade infrequently, perhaps once every 5-10 years for the average house. The amount of sales evidence varies widely in particular cases, but generally there are few sales of properties similar enough to be considered "comparable" and none of identical properties.

So instead of looking up prices in the financial press, as one would do with a share or commodity price, people interested in prices of particular property assets consult valuers who collect and interpret recent sales evidence in order to arrive at a price estimate based on interpretation of *differences* between properties.

The market has the same problem as the valuer—how to discover prices of heterogeneous assets where there are few similar transactions and many property characteristics that influence prices? For any individual property at a particular point in time, different prices are possible due to different circumstances of sale, differing buyer preferences, different buyer information sets or other factors. We may call this variation "random error" because we don't know its causes. This means that the observed prices used by valuers to infer value of a subject property by sales comparison include random variation. Po, the observed price, is equal to $P\mu+\varepsilon$, the mean of the possible price distribution, plus a random error. We do not know $P\mu$ or ε, we only know Po, the transaction price we observe.

Heterogeneity requires valuers to develop models of price differences. Instead of P(t)=P(t-1), where price of the subject property equals recent transaction prices[5], valuers have to use Psubject(t)=Pcomparable(t-1)+differences. "Differences" means the price implications, positive or negative, of the differences in hedonic characteristics between the properties. This "sales comparison price differences" regression model is mathematically equivalent to the "adjustment grid" used by American valuers (Colwell, Cannaday & Wu, 1983). Modelling price differences due to differing characteristics stems from Kevin Lancaster's notion that utility and the price people pay for complex commodities like housing or automobiles is a sum of the utility of various characteristics (Lancaster, 1966, Rosen, 1974).

Valuer's tasks therefore include:

a) Choosing which sales are best to use to infer price of a particular property.

b) Identifying price-affecting characteristics that differ between sales and subject property.

c) Estimating the dollar value of these differences for each pair-wise comparison of subject and sale.

[4] In a review of a sample of hedonic regression papers, we discovered that literally hundreds of variables have been found to be statistically significant price predictors (Kummerow and Watkins, work in progress).
[5] Examples: "Gold is trading at $325 per ounce," or "BHP shares closed at $9.90."

d) "Reconciling" to give a single price estimate, where indicated values of the subject from different adjusted comparable sales are not identical (the usual outcome).

Two different kinds of errors arise in this "valuation by modelling price differences" process. First, there is the random variation of sale prices discussed above. Second there are errors in estimating the value implications of differences between the properties. Total error is the sum of random plus adjustment errors.

If the standard deviation of a possible price distribution is σ, then the standard deviation of the means of samples "drawn" from the distribution is $\frac{\sigma}{\sqrt{n}}$, where n is the number of sales in the sample.[6] Therefore, increasing the sample size reduces the variation in sample means allowing for more precise estimates of the property value. Probabilities can be estimated because the law of large numbers states that as sample size increases, the sampling distribution of the mean becomes approximately normally distributed. The normal curve has a known probability density function.

We cannot actually get multiple observations from the possible price distribution of the subject property, so we use the adjusted sales prices of comparable properties as proxies for events (transactions) from the subject property's possible price distribution. The number of comparable sales depends on how much sales evidence can be obtained and the valuer's choice of sample size. Each adjusted sale proxies for an outcome from the possible price distribution of the subject property. Combining these indicated values of the subject allows for a more precise value estimate than if a single comparable sale had been used.

But properties are heterogeneous; they are more or less different from the subject property. So as the sample size increases, the variance, σ^2, of the sample increases. So although errors in the mean of the sampling distribution are *decreased* by increasing sample size, if the increase in variance exceeds the effects of the larger sample, the law of large numbers may not hold true. Moreover, measurement and misspecification errors in the price differences model also tend to increase as we add more comparable sales (Kummerow and Galfalvy, 2002). So there is an error trade-off and larger samples may not help us get more precise estimates.

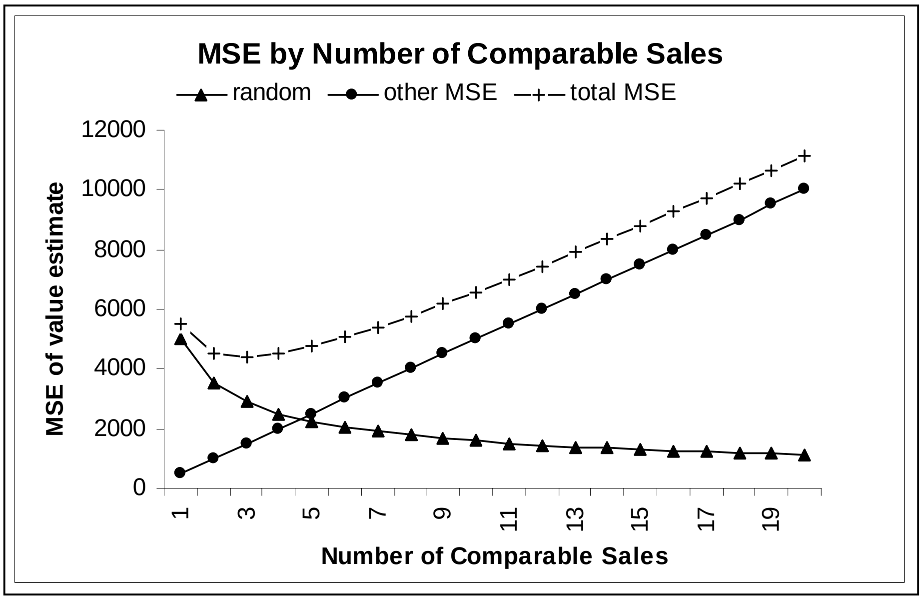
Valuers' errors in price prediction arise from both random variation in observed prices of comparable sales used as evidence and the mistakes in the valuer's model of price differences. While these two kinds of errors can be conceptualised separately, they can only be observed jointly through the differences between valuations and sale prices.

Kummerow and Galfalvy (2002) present a view that all pricing models are misspecified so there are possibly biased adjustment errors when price differences between subject and comparable sales are estimated. We argue that the error trade-off between random pricing errors in the observed sale and valuer

[6] Where a sample estimate s is substituted for σ then the denominator becomes sqrt. of n-1.

pricing model adjustment errors can lead to a "U" shaped total error distributions when errors are plotted against the number of comparable sales. (Figure 1),

Figure1 Mean square error trade-offs in valuations as sample size increases



Source: Kummerow & Galfalvy, 2002

In heterogeneous populations a "law of medium numbers" can hold, where optimum sample size varies between data sets but is usually not large. It could be that optimum sample size for minimising price prediction errors could be as small as *one* comparable sale where random errors are small and adjustment errors large. Conversely, if random errors in observed prices are large and adjustments (price differences models) accurate, then a larger number of comparable sales will produce a more precise estimate. Valuation practitioners seem to think the optimum sample size to optimise the error trade-off and minimise total mean square error (MSE) is quite small, often only three sales, as shown in figure 1.

Because the sales are not all equally comparable to the subject property, a complication is that we usually prefer to use a weighted average, reflecting the fact that some sales are better proxies (more similar to) the subject property than others. They give more weight to the "best" i.e. most similar, sales, where they are confident that the price differences model (adjustments) errors are small. Courts of law have taken the reasonable position that the "nearness" of each sale to the subject property needs to be taken into account rather than simply computing an average.

Summary so far:

- Price of a specific property at a point in time is a random variable reflecting the heterogeneity, uncertainty and limited information of buyers and sellers. Therefore, at a given moment in time, there is actually a *probability distribution of possible prices* each property might sell for.

This distribution is unobservable because we only see one event from the distribution, the actual sale price that is “drawn” from the distribution when the property sells. Kummerow (2000) discusses “clues” to the nature of this unobservable possible price probability distribution.

- Valuers estimate models of the dollar implications of differences between properties, inferring likely sale prices of a subject property from samples of comparable properties’ observed prices adjusted to reflect differences between the sale and subject properties.
- There is a trade-off between random errors that decrease as sample size increases the precision of sampling distribution estimates of the mean of the possible price distribution versus price adjustment errors that tend to increase with sample size.
- Prediction errors provide the test of sample selection and price difference models because calculated statistics may be biased by misspecification and measurement errors.

In teaching valuation, therefore, key concepts are:

Possible price distribution. A probability distribution showing the relative probability of different prices being revealed in a sale of the subject property at a particular time.

Pricing model. A parsimonious representation of the market’s pricing process used to “adjust” sales evidence to obtain an “indicated value” of the subject property.

Error analysis. Errors in predicted prices provide a means for making estimates of variance of possible price distributions and the evaluating the accuracy of pricing models.

Literature

Professor Richard Ratcliff proposed a restatement of valuation theory emphasising that valuation is a prediction of human behaviour under uncertainty. He discussed “transaction zones” pointing out that depending on negotiation skills, any one of a range of prices might emerge from a sale process. Ratcliff drew bell shaped probability distributions of property prices. (Ratcliff, 1972) Maurice Squirell expanded on Ratcliff’s notion of uncertainty in property prices (Squirell, 1985).

Many papers in the academic literature present hedonic price models. These represent the insight that people buy a bundle of characteristics of properties, not a simple, one-dimensional source of utility. Many factors influence the prices people pay for real estate. These models are written as:

Price=coefficients *characteristics + error

The coefficients are weights—dollars per unit of characteristic. The characteristics are features of the property that have an effect on utility to buyers. The jargon term "hedonic" means pleasure in Greek. The theory of hedonic pricing is that people's willingness to pay reflects their valuation of bundles of hedonic characteristics, rather than a single one-dimensional generic good. An analogy can be made to a basket of groceries; the value of the basket is the sum of the values of the milk, meat, eggs, etc. in the basket (Lessinger, 1969). In the hedonic pricing model, the coefficient is like the price per litre of milk and the number of litres of milk is the quantity of the hedonic characteristic. For example, part of the value of a house might be due to its size, so we might multiply \$500/m² times 200 square meters of floor area to get \$100,000 as the contribution of the "size" hedonic characteristic. Then we might add more value for the lot area, views, something for the nice kitchen and so on to get the total value estimate as a function of the amount of hedonic characteristics offered by the house.

The academic literature includes thousands of hedonic pricing models where price as a function of hedonic characteristics is estimated by multivariate regression. Since hundreds of variables have been found to be significant in various studies, it is clear that these markets are complex and it is no wonder both buyers and valuers express some uncertainty about prices. Interaction effects and non-linear relationships between prices and hedonic variables complicate the issues. Pre-test biases, misspecification and measurement errors are common in published models, leading to large standard errors and poor out of sample prediction.

Academic authors have mostly used pricing models in levels rather than differences, estimating prices directly from property characteristics, rather than including comparable sales prices in the model. Usually a large but not very homogeneous sample of sales prices is used to estimate best fitting coefficients in an hedonic price model.

The equation is Ps=ΣbiXi where Xi are property characteristics like size, age, etc. and bi are coefficients or weights. The sales comparison model is, instead, Ps-Po=Σai(Xsi-Xoi), the price differences model, estimated from a small sample of quite similar or "comparable" property sales (Colwell, Cannaday & Wu, 1983). Ps is the subject property price, Po an observed comparable property sale price. I use ai for the coefficients on the differences in characteristics, rather than the same bi as in the regression model because these coefficients are not the same. The bi are something like the average contribution to price of, say, an extra square meter of house size, while the ai are more like a marginal price, the value of a square meter of house size difference. Normally these coefficients would differ because price is not a linear function of most property characteristics.

Colwell, Cannaday & Wu, 1983, pointed out that the price difference model is formally mathematically identical to the "adjustment grid" method used by practicing appraisers in the U.S.A. Subtracting the observed sale price from both sides makes it explicit that this is a model for estimating price differences:

Ps-Po=price difference between subject and sale

The “trick” or insight of this sales comparison approach is that it sidesteps the necessity to estimate price effects of most of the hedonic characteristics since they are the same in the two houses and this similarity applies to both included and omitted variables.

The valuer has to identify relevant points of difference between the properties (the X’s, the characteristics that make for price differences) and estimate “aj,” a vector of price effects of those few characteristics that differ between the two properties. The number of characteristics in the price differences model is fewer than the number in the regression model. Pace refers to using bj regression coefficients in a sales comparison price differences model as the “plug in” method (Pace, Sirmans & Slawson, 2002), but finds they are not the most efficient estimates.

How well sales comparison “works” is “data dependent.” The “closeness” of comparable sales to the subject property varies. “Closeness” here means not just geographical nearness, but rather nearness or similarity in hedonic characteristics space. As every valuer has experienced, in some cases there are plenty of good comparable sales to use, and in other cases there are fewer and less similar cases, perhaps even none close enough to allow easy modelling of price differences. So errors in price estimates vary on a case-by-case basis, depending on the quality of the sales evidence. The random errors in prices themselves can also be larger or smaller depending on the uniformity of the market’s opinion of a property.

In a price differences model, many variables that might be important in pricing can be ignored because they do not differ between subject and observed properties. Size of the house, for example, generally shows up as important in regression models of property prices---big houses sell for more than small ones. But in a sales comparison model, if all the comparables and the subject are of such similar sizes that the market cannot notice the differences, size might not matter. Sales comparison models focus only on points of difference between subject and sales and the less important the differences the better.

The quality of a sales comparison model will depend on how completely and correctly the valuer has inventoried the points of difference that matter in pricing. And secondly on how well she has estimated the price effects of points of difference in this particular context. The effect of a variable might be considerably different in a small subsample of comparable sales compared to the average effect across the whole population of sales.

Despite many attempts by academic authors to value properties using regression, a review by Lenz and Wang (1998) pointed out the large standard errors typical of regression pricing models.

Real Estate Valuation Theory, edited by Ko Wang and Marvin Wolverton (Kluwer Academic Publishers, 2002), includes 18 articles by academic authors covering a range of topics in valuation theory and methods. Several articles apply statistical methods to more disaggregated data, thereby coming closer to mimicking the sales comparison method used by practitioners

A "hot topic" in academic empirical work on valuation is spatial statistics, where "spatial" can means geographical space or alternatively "near neighbours" in hedonic characteristics space. Isakson (2001) proposed a way of calculating distances in multi-dimensional characteristics space to identify "near neighbours." Dilmore, Graaskamp and Robbins had earlier developed a method for calculating distances (in multivariate hedonic characteristics space) and identify best comparable sales.

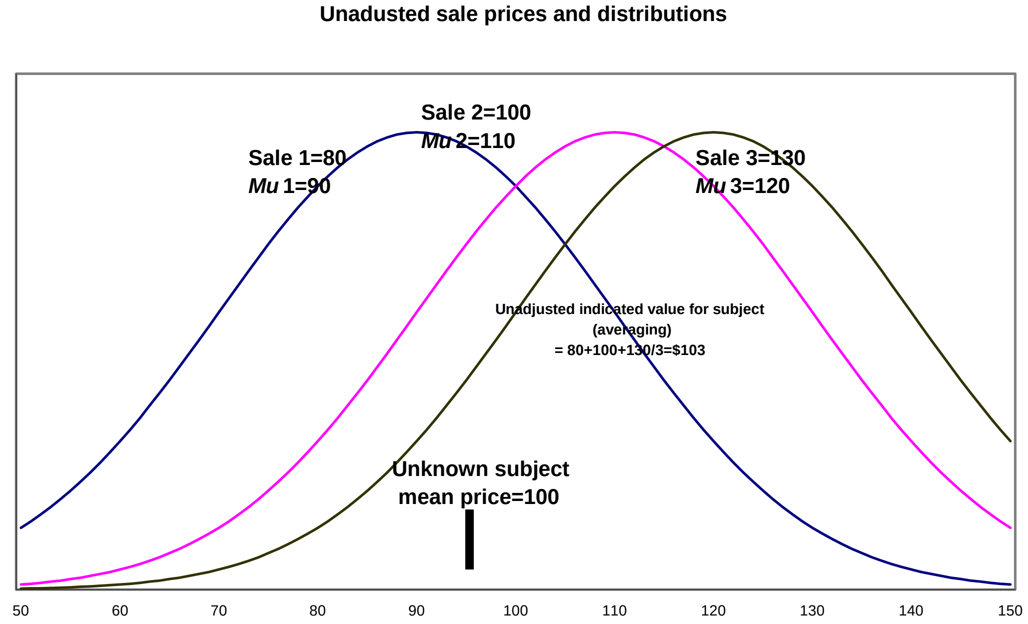
Watkins and others have shown that estimating prices for submarkets is more accurate than using larger aggregations of sales. The sales comparison approach of Colwell, Cannaday & Wu provides a rationale for even more disaggregation. Kummerow and Galfalvy's error trade-offs story suggests disaggregating to a few comparable sales, as is customary in valuation practice.

Pace, Sirmans & Slawson (PS&S 2002) begin an article by remarking that "statistically challenged" practitioners usually produce more accurate valuations than those of learned professors who use high-powered econometrics. PS& S therefore proceed by using a "grid estimator" quite similar to sales comparison as practiced by valuers. The Colwell, Cannaday & Wu "trick" of using the price of a comparable sale as a proxy for an unknown complex, unidentifiable pricing model is beginning to be recognised by academic researchers as a more accurate method than regression when data are heterogeneous. Using "distance in hedonic characteristics space" as the comparable sale selection criterion is an objective way of selecting comparable sales.

Tax assessors have also done a great deal of work on statistical or database methods of valuation and on valuation quality control. The IAAO's textbook on *Mass Appraisal of Real Property* (Gloudemans, 1999) proposes the appraisal/sale price ratio (A/S) and the coefficient of dispersion as measures of appraisal bias and precision respectively. Assessors' work on quality control provides useful tools for all valuers.

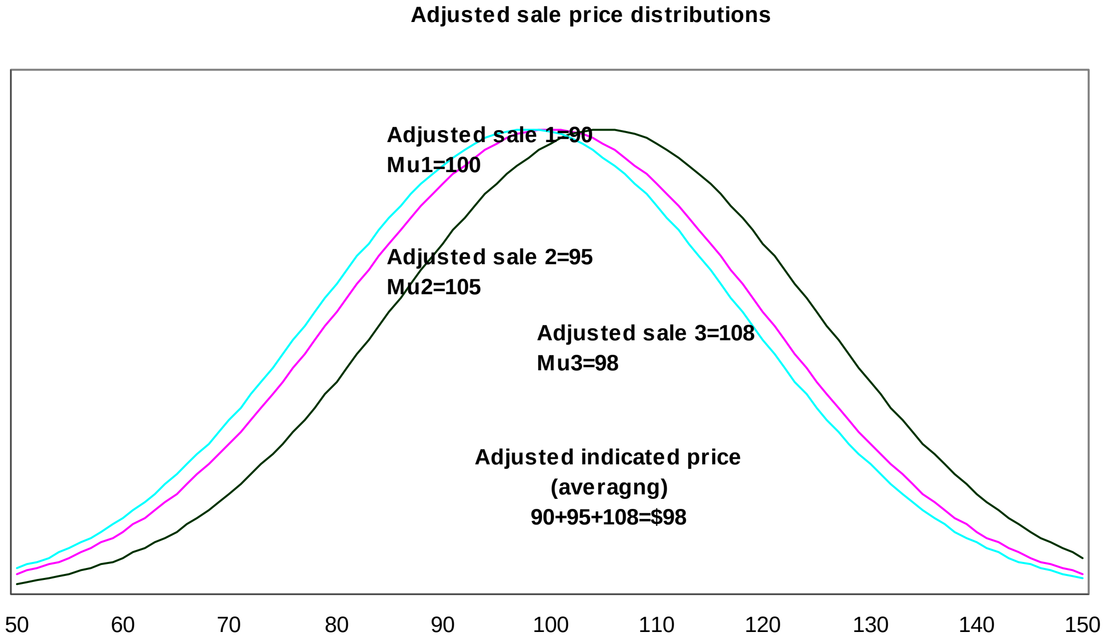
Figure 1, copied from "Thinking Statistically about Valuations," (Kummerow, 2000) shows three observed prices and the possible price distributions of three comparable sales. Adjusting these sales to reflect differences in hedonic characteristics between the comparables and a subject property leads to figure 2, where the three adjusted distributions now are shifted to "indicate" the possible price distribution of the subject property.[7] We observe only the sales prices. The sketches of distributions, however, make the point that the observed prices are drawn from distributions and that sales probably do not occur at the expectation of the unobserved possible price distribution. After adjustments using a price differences model, the three sales come closer to "telling the same story" about the subject property price (Figure 3). But the three "indicated values" for the subject are still not identical. A valuer would "reconcile" the three indicated values, thereby merging them into a single estimate of the mean of the subject property's possible price distribution. The subject price is a probability distribution of a random variable. The valuation should therefore be a density forecast, not a point forecast.

Figure 2 Three sales and their possible price distributions



[7] What we have done here is use a price differences model to estimate the value of the differences between the properties and added (if the sale was inferior) or subtracted (if sale was superior) to transform the sale price into an indicated price for the subject.

Figure 3 Three "indicated values" for the subject after "adjustments" to the observed sale prices reflecting differences from the subject property.



In another *Appraisal Journal* article (Kummerow, 2002) I proposed the following four-part definition of value:

- Estimates of the moments of the subject property's possible sale price distribution
- Estimates of errors in the estimates and diagnostic tests.
- Forecasts of the stability of the estimates over a relevant period.
- Statements of explicit assumptions about the circumstances of sale that may influence the possible price distribution including legal rights valued, date of sale, method of sale, time on the market, finance, probable uses of the property valued, probable buyers and motives/knowledge of buyers and sellers.

This definition seems clearer than court mandated verbal definitions with their assumptions about prudent and informed buyers, etc. And this definition connects valuation to useful statistical concepts and methods.

Theory

There are four points to list as foundation valuation theory:

1) Value equals discounted expected future benefits of ownership

2) Value is proportional to expected utility of various property characteristics, that is, we buy bundles of hedonic characteristics. (See Rosen 1974 for discussion.)
3) Prices are revealed in market transactions. For over a hundred years, economists have accepted Alfred Marshall's theory of buyers and sellers interacting "like two blades of a scissors." Absent a sale, or somebody talking about a potential sale, there is no way to estimate value. Prices emerge from markets, they are not inherent in the property itself. An asset is worth what someone will pay for it on the day.
4) Prices tend to adjust towards equilibrium where supply and demand would be in balance and prices unchanging, but this process takes time and markets are normally not at equilibrium.

Practice

An American form appraisal called the Uniform Residential Appraisal Report (FANNIE MAE Form 1004) lists the following hedonic characteristics of houses that warrant price adjustments:

1. Sales or financing concessions
2. Date of sale/time
3. Location*
4. Leasehold/fee simple
5. Site/view*
6. Design and appeal*
7. Quality of construction*
8. Age
9. Condition*
10. Above-grade room count/gross living area
11. Basement and finished rooms below grade
12. Functional utility*
13. Heating/cooling
14. Energy-efficient items
15. Garage/carport
16. Porch, patio, deck, fireplace(s), etc
17. Fence, pool, etc.

*Qualitative items clearly requiring subjective evaluation by the appraiser.

Source: http://www.dearborn.com/download/frea8e/Chapter10.htm

These are used to create an "adjustment grid" whereby the amount of each characteristic in the subject property is compared with the amount in a comparable property that has sold recently. These differences are then multiplied by an estimate of the price of the characteristic to get an estimate of the value difference between the two properties that is attributable to the difference in the hedonic characteristic.

Example: Suppose a subject property has a floor area of 200 m^2 while a similar property that sold for $300,000 has 220 m^2. If an extra square meter of space is worth $500, then the difference in price should be 200-220=-20 * $500= -$10,000. Subtract $10,000 from $300,000 to get $290,000 as an estimate of the probable selling price of the subject property. The subject property is smaller, therefore inferior, and therefore should sell for less.

As mentioned above, empirical work has identified hundreds of significant hedonic characteristics and so the list of "what matters" varies between properties. The valuer's job is to understand which characteristics increase or decrease buyers' willingness to pay in the case of a particular property and by how much.

An important question is how to choose comparable sales? The answer is to choose sales that result in the smallest errors in predicting the price of the subject property. How to operationalise that ideal may not be clear. To minimise adjustment errors, one would want to pick sales that either have very small adjustments so even a big error in the adjustment would not make much difference or, if adjustments are large, to use sales where the amount of the adjustment can be estimated precisely.[8]

Valuers in Australia do not generally use explicit adjustments for particular property hedonic characteristics. Instead of the American adjustment grid they prefer to locate sales that are slightly superior and slightly inferior to "bracket" the subject property and then make an overall estimate of how much more or less the prices of the sales will be compared to the subject. While this may seem to be inferior to the more systematic and transparent U.S. method, in fact this "gestalt" or overall pattern method works fairly well and has some advantages. The valuer is not restricted to a set list of characteristics, so anything that matters can be considered. Differences are not constrained to any simple functional form and interaction effects can be considered.[9] A weakness that because this overall comparison method relies so much on judgement, different valuers can and do come to different conclusions.[10] It is hard to write down any convincing method whereby the valuer could prove his conclusions from the evidence. Vandell criticised practitioners' "ad hoc" methods that could "allow bias to enter." (Vandell, 1991)

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[8] It is also reasonable to exclude non-arms length transactions and other outliers from the sample.

[9] Colwell, Cannaday & Wu, 1983 point out that summing dollar adjustments in a grid is an additive model of price differences, while percentage adjustments imply a log transformation and a multiplicative model. But in the real world, many other functional forms, threshold effects and interactions are possible.

[10] In Western Australia a recent Royal Commission investigated valuer culpability in a "mortgage brokers scandal" that cost mostly elderly investors AUS$150 million in losses. The "Temby Report" of the Royal Commission was highly critical of valuers, listing a number of practices that distorted values relied on by investors.

Three approaches to value

The theory presented above leads to the conclusion that all valuations require reference to market transactions that reflect supply and demand conditions as well as buyers' and sellers' expectations of future benefits of ownership. However, valuation has traditionally been presented to students in the form of "three approaches to value," namely sales comparison, cost and income

Sales Comparison can lead to mispricing

Because current markets can be far out of equilibrium during bubbles or busts, recent transactions can paint too rosy or pessimistic a picture of longer-term outcomes. Current transactions may represent *mispricing* in the sense that a knowledgeable person would have good grounds to forecast future price increases or decreases.

Current market prices of shares are found by the trivial exercise of consulting the financial pages of a newspaper, calling a broker, logging into a website or watching TV news. The current price is not, therefore, the issue that financial analysts (the valuers of share market assets) consider. Instead they give their attention to a search for mis-priced assets and to forecasting future price movements. They try to help their clients make money by offering expert advice about asset values over a future period. In property markets, oddly, experts leave most of this kind of thinking to the investors. Few people consult a valuer to ask them to find mis-priced property assets or to predict investment outcomes. But there is a trend for valuers to be asked for this more sophisticated advice (Appraisal Institute White Paper, 1999). Many clients may wish to better understand potential risk and return in making a real estate decision.

Cost does not equal market price very often

Economic theory says in the long run, cost should be related to value. Something that takes a week to build should cost less than something that takes a year to build. Supply will adjust until price=cost at equilibrium. But property markets are seldom at equilibrium so, in general, cost does not equal price. Supply adjustments involve long time lags. If market prices are above equilibrium, there is a potential developer profit, which can be quite substantial. $1 of land plus $1 of building might equal $3 of market value. In a property bust, cost may be much greater than current prices. In Perth's oversupplied market in 1994, new office buildings sold for as little as 1/3 of their cost. Supply and demand rule short-run prices. Sales transactions are necessary to reveal the current relationship of costs to prices. The tendency will be for markets to adjust towards equilibrium—if prices are above costs, then new supply will tend to be created and prices will fall. If prices are below costs, construction will cease until demand increases or buildings are removed from the stock. But because adjustments are slow, at any given time, the market is likely to be out of equilibrium.

The income approach requires sales to find discount rates

Finance theory reached consensus by the 1920s that the value of an asset depends on the discounted expected future benefits of ownership. Any finance textbook has a passage something like “the value of any asset should be a function of three variables: how much the asset generates in cash flows, when these cash flows are expected to occur, and what uncertainty is associated with these cash flows. Discounted cash flow valuation brings all three variables together by computing the value of any asset to be the present value of its expected future cash flows.” (Damodaran, 2001) This tool of DCF (discounted cash flow) analysis finds ready application in property markets because cash flows are often somewhat forecastable due to long term rental contracts or patterns in past rents and operating expenses in a particular market segment. Nevertheless, the discount rate emerges from market’s assessments of the risks of projects and supply and demand for investment funds in the capital markets. Again, sales comparison is the fundamental tool used to value properties by the income approach. Discount rates are revealed by sales.

As noted above, it is odd that in property markets, valuers have been trained to avoid forecasting future prices (future cash flows), when in finance markets this is the key to analysts’ evaluation of investments. Property valuers relegate themselves to the relatively unrewarding task of substituting for the stock price ticker or financial press. This passive reporting of recent transactions, without opinions about whether the investment is properly priced or any bets about whether it is worth owning, in terms of future performance, adds less value than share market “valuations.” As the property industry continues its integration with the wider capital markets, surely valuers will have to adopt the standard finance view of asset values and devote more attention to offering market analysis and going beyond providing mere current price estimates.

Two methods

Valuation methods fall into two main categories: Objective and subjective. Another way to express this classification of methods would be “science” versus “art.”

Objective methods: Valuation as science

Science is a rational paradigm of inquiry where conclusions are based on evidence observable to others. Objective observers, looking at the same evidence should be able to replicate scientific findings.

Nearly all academic writers on valuation methods and probably a strong majority of practicing valuers would say valuation aims to follow the scientific paradigm. That is, conclusions are supposed to be based on evidence. Courts are usually unimpressed by opinions not supported by evidence. Different valuers, who are supposed as professionals to operate without bias, should come to similar conclusions. We know that client pressures result in valuers for landlords proposing higher rent reviews than valuers for tenants, but this is a corruption of the ideal scientific process.

Valuation as science leads to setting out methods and standards and performance criteria. Scientists write “protocols” that is, step-by-step descriptions of methods used in a particular procedure to ensure quality control.

Subjective methods: Valuation as art

Science is all very well when there is sufficient sales evidence to make a good case for a value. But what about cases where sales evidence is lacking or confusing?

An experienced, highly respected Australian valuer cites a case in point. In a rent review during a property oversupply cycle when landlords were offering big leasing incentives to try to fill empty buildings, he located recent evidence for net effective rents ranging from negative[11] to \$115 m^2. He chose, after considering this evidence and other factors, to review the rent to \$125/m^2, a figure *outside* the range of evidence. He believed the rental evidence to reflect temporary market conditions that would not continue through the lease term. This is valuation as art, as a creative act. This valuer did not *measure* the rental value so much as *create* rental value when the market was clearly confused about what the rents should be.

This example, in exaggerated form, exposes a problem valuers face in nearly all valuations: Available evidence insufficient to allow drawing unambiguous conclusions.

Once we accept that samples are heterogeneous, then we also accept that any estimate based on statistics—whether by descriptive statistics or regression or whatever method—will report average responses that may not be a good representation of the pricing process for a specific property. The value of a feature (hedonic characteristic) in a particular house may differ from the average. Estimates of effects from small homogeneous samples can vary greatly from estimates from larger samples. (Kummerow & Galfalvy, 2002).

There are always omitted variables. Hundreds of variables can affect prices and many of these are never measured. Fundamental issues like buyer incomes and whether buyers are “informed” as specified in the standard market value definitions are seldom investigated. Omitting variables can result in biases in the estimates of effects of included variables.

Measurement errors in the data can be another problem that reduces the precision of statistical estimates. Many attributes of property that buyers respond to are intangible, emotional, aesthetic issues like the quality of views or the prestige of an address. These are obviously hard to quantify. Even simple “countable” property characteristics, like number of bedrooms or bathrooms may mask huge quality differences. Does one bedroom mean a huge room with walk in closets, an adjoining spa and views of the mountains from the balcony or does it mean a tiny cubicle lit with a bare light bulb, worn out carpet and peeling paint? Both

[11] Negative net effective rents could be rational for a landlord concerned with getting a tenant into the building to help carry fixed operating costs. Leasing incentives would have played a big part in the rent cuts.

could show up as “one bedroom” in “quantitative” data used to estimate prices. This is a form of measurement error that obviously influences price estimates.

If we look carefully at the theory behind hedonic modelling, we find a serious identification problem. Because so many variables vary simultaneously, it is not possible to sort out which variables are responsible for price effects without making unrealistic assumptions. (Rosen, 1974, Kummerow and Watkins, work in progress).

The pricing models used by consumers may be relatively complex, non-linear and include many interaction terms. Buyers take account of many factors, including some they may not even be consciously aware of. Buyer preferences are not stable, but may evolve as a search process goes forward, new information is revealed and market conditions change. There is too much going on, too many interactions for individual effects to be sorted out in simple models. With ten data points, twenty variables and non-linear relationships, the correct pricing model cannot be identified.

So valuers guess at prices with errors and uncertainty. They create approximate value estimates by art and subjective evaluation as well as measuring values through analysis of objective price evidence. This is not much different from what buyers and sellers do in markets. Everyone faces decisions that involve complexity and uncertainty.

One of the main tools in the social sciences, according to the German sociologist Max Weber, is *verstehen*, meaning understanding. We may be able to make a reasonable guess about how a market would evaluate a property because we are culturally similar to buyers in many respects. If it seems good to the valuer it may also seem good to the market. Subjective opinions and “gut feel” could very well produce better value estimates than “scientific, objective” statistical procedures based on flawed data and misspecified models. There is no unambiguous statistical recipe to arrive at a model specification. Choices have to be made even in “objective” methods.

So the two basic valuation methods are: a) Science, b) Art. In the former the valuer reasons from evidence using quantitative methods. In the latter the valuer creates a value by subjective opinion based on experience—an educated guess.

Modern societies are dedicated to rationality rather than superstition. We need to start with data and go as far as possible with data, but no farther. Pretending to get a precise quantitative answer from a quantitative method that is flawed is just another form of superstition. Once the data have told us all they have to say, we may need to rely on subjective opinion or “experience” to fill in the holes in the story. “Let the data speak,” an econometrician joked, but then “tell the data to shut up.”

Valuers should not offer unsubstantiated opinions. Readers of valuations should be able to follow the valuer’s train of thought. They should be able to check or replicate how the value was arrived at. The evidence should be presented. The method of analysing the evidence should be transparent. No black boxes from which a value emerges. The valuer should get the details right and dig out the relevant issues and present as thorough and complete a case as allowed for by data, the time available and the fee.

We can check on both objective and subjective methods by comparing value estimates to subsequent sale prices. Random errors could make these differ in a particular case, but over a reasonable sample of valuations, the spread in errors of the valuations should approximately equal the spread of errors in the possible price distribution. And the mean of the valuations should reveal no systematic over or under estimates.

Nearly all academic work on valuation regards property value as a random variable. Because it is a random process, we cannot predict in advance in any particular case what the error will be in a single sale. As in a throw of dice or cutting a deck of cards, we may know something about the probability distribution of possible outcomes, but we don't know which number will come up in a particular throw or draw.

Confusion arises from not recognising random variation. For example, with the random variation or probability distribution idea, one would not talk about "the true value" of a property. Many prices are possible with varying probabilities of occurrence. Any single paired sale comparison can be misleading due to random error. This statistical view of value requires coping with variation by using summary statistics of samples as price estimates, rather than single observations.

But the law of large numbers does not hold in most real estate data. The reason is that as sample size increases, so does the variance of the sample, because properties are heterogeneous. Moreover, measurement errors and misspecification errors (meaning omitted variables) tend to increase with sample size. So there is a trade-off, essentially between the random errors in the observed prices that can be "averaged away" by increasing sample size versus the increasing adjustment errors required to make more and more disparate sales proxy for draws from the possible price distribution of the subject property. This trade-off plus the cost of looking at more comparables are the main reasons why valuers use only a few sales in sales comparison methods. But even using a few sales confirms whether the prices indicated by similar sales provide consistent estimates. We rely on efficient markets—small random errors by buyers—to allow reasonably good estimates, even with small samples.

The weakness of sales comparison, from a statistical point of view is that estimates from small samples can be unstable. There is no blanket statement that can be made regarding how big a sample is big enough because it depends on the variance of the population and the degree of precision desired in the estimate.

But we can look at estimates "recursively", that is, plot how estimates change as sample size increases from 1 to n (the maximum feasible sample) to explore pricing model reliability. We need to use a large enough sample to achieve relatively stable price estimates. "Large enough" might be one comparable sale, if the market is efficient and random errors are small. The "law of medium numbers" arising from the error trade-offs story says that samples can be too big in that precision of estimates will deteriorate when sample variance and adjustment errors increase as the sample becomes more heterogeneous. Prediction errors can be used to approximately identify the optimal sample size.

Estimates may not be stable because the response to a variable in a particular subset of the data (say the bigger houses, for example) may differ from the response in other cases. This is a problem with reality rather than with models. The real world is a bit messier than it should be for convenient modelling.

Teaching valuation

To teach valuation start with concepts from statistics:

1. Random variable
2. Probability distribution
3. Moments of a probability distribution (mean, standard deviation)
4. Sampling distribution
5. Confidence intervals
6. Measurement and misspecification errors
7. Prediction errors

Add concepts from the hedonic literature:

1. Hedonic model (Lancaster, 1966)
2. Hedonic price differences model (Colwell, Cannaday & Wu, 1983)
3. Error trade-off, law of medium numbers (Kummerow and Galfalvy, 2002)
4. Grid estimators, spatial autocorrelation (Pace, et al. 2002)
5. Prediction errors, Valuation/Price ratio and coefficient of dispersion to evaluate pricing models and sample selection (Gloudemans, 1999)

From these foundations students can go on forever learning more sophisticated statistical estimators or just learning the local market—acquiring experience about what matters to buyers and sellers.

An objective valuation algorithm

The following is an attempt to set out an algorithm or protocol for producing an objective valuation from a set of sales data. An example in an Appendix shows how this set of steps can be used to estimate the first and second moments of the possible price distribution.

1. **Define a submarket**. Note that this is subjective, but we can iterate back and redefine the submarket if the end result—the valuations—fail to predict prices well. Data should be recent sales from a defined geographical area, perhaps a few similar suburbs (Lusht & Pugh, 1986) or an area where properties are similar. Examples of "screens" for defining submarkets:

- 3 bedroom, 2 bath homes
- Industrial buildings 10-20 years old, size between 2000-5000 m2, and truss height at least 5 meters
- Class A CBD office buildings between 10000 and 30000 m2 NLA.
- Homes in the $300,000-400,000 price range.

The subject property should be near the midpoint of the submarket. In the above examples it should be located in the heart of the geographical area chosen, in a typical area, it should be a 3x2 home, or a 3500 m² industrial building, a 20,000 m² class A office, or a $350,000 home respectively. As many comparable sales in the submarket chosen should be “above” as “below” the subject.

2. **“Clean” the data** to eliminate obvious mistakes and outliers, taking care not to bias the data. If the top price in the data set is eliminated, also discard the bottom price, to maintain the balance on each side of the subject property. A sample of 30 to 300 (more or less) sales should be chosen from the submarket, again editing out sales in an even handed fashion to “bracket” the subject property into the middle of the sample.
3. **Inventory points of difference** between subject and sales. Why do prices vary in this sample? What kind of price differences model would be adequate to capture (explain) most of the price differences? This exercise almost certainly requires property inspections and market knowledge gained through discussions with agents, buyers and sellers and experience. Many locational factors, for example, are only noticed by going to the site and looking around. Points of difference like access, views, condition, appeal of the design, and spatial relationships with other surrounding properties, etc. are not usually identified without inspections.
4. **Model price differenences**. Select a short list of the most important points of difference (in terms of effects on values) and estimate their effects by means of graphs, summary statistics, regression estimates or other techniques. If subjective estimates are the only way to come up with a number, write down the number and some justification for it. This is not a model of prices, but rather of price differences—hopefully the main factors influencing prices (neighbourhood, property size, etc.) do not vary in this relatively uniform submarket. What we are after is the remaining few issues that explain the variation within a fairly uniform subset of sales.
5. **Adjust the sales in the submarket. Use the differences model estimates to select a smaller subset of comparable sales based on minimum adjustment criteria.** A standard used in the U.S. is that a property can be used as a comparable sale if the total absolute value of adjustments is not more than 25% of the sale price and the net adjustments (sum of positive and negative adjustments) is not more than 15% of the sale price. These screens can be tightened or loosened to reflect available data. Discard any outliers or sales that differ in important respects from the subject property, taking care not to retain a biased sample. From 5 to 10 comparables should remain after this second screening.

6. **Value each of the comparables using from 1 to all of the remaining comparables**. That is, if there are ten properties, do nine valuations of each property, using from 1 to 9 comparables in each case. This will give a total of 90 valuations for which the actual prices are known. Calculate prediction errors for all of these valuations and look at the distribution of errors histogram. Calculate the valuation/sale price ratio and coefficient of dispersion. Calculate summary statistics of the prediction errors. Graph or tabulate how prediction errors vary with sample size. This will allow you to choose an error minimising sample size to use in valuing the presumably similar subject property. While this may sound too time consuming for practical implementation, in fact, these multiple valuations can be implemented in a few seconds using the speed of a computer and Excel macros or other programs to perform repetitive calculations. Within a few years it will probably be possible to buy programs to do all this with only the data and a few input assumptions entered by the valuer.
7. **Value the subject property, using from 1 to all of the comparable sales**. So if there are 10 comparables, you will value the subject 10 times. Choose as your preferred value estimate the one from the sample size that showed minimum error in the tests in the previous step, but report all ten indicated values to demonstrate to the client the stability or instability of the estimate. Report an estimate of the variation in the price expectation estimate, based on the variation in the indicated valuates from the different comparables and the errors found in step 6 above. Report the variation to the client.
8. **Iterate, modifying any step above and repeating the process to see if the sample or model of price differences can be improved with respect to the price prediction test** in step 6 and whether model and price estimates are robust across changes in samples of sales and in different price differences models. Generate statistics on model performance—valuation to price ratios and coefficient of dispersion. These are measures of valuation bias and precision.
9. **In an appendix list all sales discarded and reasons for not using them as comparables, the rationale for choice of submarket, the price differences model and how it was estimated.** This is necessary to help keep valuers honest.

Other issues might be added. Before we come to the above steps, we need a protocol for data collection that deals with selection of relevant variables and measurement of their values for a set of properties.

If we select the wrong sample of sales, we will obtain biased results since responses differ in different parts of the sample space. This poses a circular reference or chicken and egg problem. We need a sample to get the right model and the model to get the right sample. If the model predicts poorly, the problem could be with either the sample or the model, so iteration, repeating the above steps might be necessary, modifying the data and/or the model.

In the messy world of real estate, everything becomes data dependent with responses to hedonic models varying between samples and salient variables differing between samples. Pricing models as well as parameter estimates vary as the sample changes.

The valuation firm of the future

In the last decade, faxes, the Internet, database technology, personal computers, cell phones and other information and communications technology has perhaps doubled the productivity of property valuers. Where four residential valuations per day were the norm in the 1980s, perhaps double that number might be expected now. And fees have been under pressure as fewer valuers can complete more valuations. At the same time, there has been increasing recognition that valuers' expertise can add value to businesses in more ways. Real estate has become more professionalised. Trained and experienced property professionals have been rewarded. (Appraisal Institute, 1999)

How much farther will the pressure on valuation employment and fees go as technology develops? Will valuers' jobs be taken over by a piece of computer software or some actuarial geek who says that on average, the client doesn't need valuations? Or, more optimistically, will new technology allow valuers to get all their work done by midday, so they will have afternoons free for golf? Or will the more sophisticated valuers become more like the highly paid share market analysts whose version of "valuation" concentrates on seeking undervalued shares and forecasting which companies will provide above average returns? Can we look ahead and speculate about the future of valuation businesses, job descriptions and incomes? Kelley Pace (2002), argues that valuers should concentrate more on data collection and identifying which variables matter to markets in the pricing of particular properties.

The valuation firm of the future may become a team effort:

- A statistical and database rocket scientist cleaning data, checking prediction errors, designing algorithms for automated valuation models (AVMs).
- A market researcher, information collector who spends time making sure the data used in valuing properties is relevant, complete, and accurate, and that the variables used are actually those used by buyers in the market.
- A client interface expert whose job is to market diverse valuation products to clients and ensure that clients understand how to use property market information to add value to their businesses. Valuers may serve more in long-term client relationships, offering ongoing advice as consultants, rather than as vendors of simple price estimates.
- And, there will still be a role for the traditional valuer whose subjective judgement and market knowledge will help transform data and quantitative model results into value estimates for particular properties.

So there may be at least four specialised roles in future valuation firms. This suggests a competitive advantage for larger firms that can assemble large databases and afford the specialised expertise to maintain, improve, analyse and market property market information. Overall, as in farming and manufacturing, we can probably expect employment to grow less quickly than incomes and output as efficiency and firm size increases. Innovation makes society better off regardless of the fate of individuals and firms who may be winners or losers in this process of creative destruction in an evolving economy driven by technology change.

Conclusion

Sales databases, fast hardware and software that make it convenient to do repetitive calculations enable valuers to implement more statistically valid valuation methods. The method used to value the subject can also be used to value the comparables and to value properties repeatedly using different price differences models and sample sizes. Prediction errors from the larger sample of valuations from various adjustment methods and choices of comparable sales will allow estimates of the variance of value estimates and suggest ways to improve the pricing model.

Valuers then will be able to report not only an estimate of the central tendency of the possible price distribution (the traditional valuation product) but also credible estimates of the variance of errors in predicted prices, an issue of interest to the client concerned about risk and investment returns. The client will receive more information about the quality of the valuation and the risks she faces. Once templates to perform the repetitive calculations are developed, it will not require more time for the valuer to offer these improved products.

Appendix

Error Analysis Example

Introduction

Automated valuation methods (AVMs) are being used as a check method for traditional valuations.[12] For some purposes (tax assessment, residential lending with loan to value ratio 80% or less), AVMs may eventually suffice as the primary valuation.

The following example focuses on finding estimates for the first two moments (mean and standard deviation) of a subject property possible price distribution. It does not explore optimum sample size or price difference model specification issues, but automated methods could be designed to look at those issues as well.

Example

Subject property

A home was selected from a set of sales data to serve as the subject property. In normal practice, the client would select the subject.

Geographical area complete list of sales

I began with a sample of 131 sales from 1998/1999 from Ballajura, an average "mortgage belt" suburb in the eastern Perth metropolitan region.

Data cleaning, initial regression model estimation and submarket sample selection

I edited the data in two ways.

- First I sorted and eliminated the five biggest and smallest houses.
- Then I regressed price on a set of hedonic characteristics: house size, lot area, house age, number of bedrooms, number of bathrooms, a year of sale dummy and a dummy corresponding to map grid coordinates (dividing the suburb into two geographical areas). T-statistics of five variables were greater than 2, number of bedrooms and number of bathrooms t-stats less than 1.
- I eliminated eleven sales, where the standardised residual was greater than 1.5. This left me with 110 "typical sales."

[12] By U.S. mortgage insurers and Fannie Mae in the secondary mortgage market.

- These remaining sales (n=110) were then used to re-estimate coefficients for the 7 hedonic variables.
- Calculating a price estimate for the subject property from this regression model confirmed that the subject estimated price falls near the middle of the submarket sample price range. If that were not the case, the sample would be modified to place the subject near the middle of the submarket sample price range.

The details of how I did this are open to argument, but the general idea is to get a reasonably representative set of sales with the subject price likely to fall near the middle of the range of their prices.

Screening to select closely matched comparable sales

Matching or nearly matching the seven hedonic characteristics extracted six comparables. Table A1 shows the relatively close match between the hedonic characteristics of the six sales and the subject property.

Table A1 Subject and comparable sales hedonic characteristics

	SALEPRICE	Yeardum	AREAHSE	LANDAREA	HSEAGE	BEDS	BATHS	Locdum
1	131000	0	145	530	3	4	3	1
2	129000	0	155	620	4	4	3	1
3	128000	0	133	548	4	3	3	1
4	118000	0	165	534	4	4	3	1
5	130000	0	129	637	3	4	3	1
6	160000	0	172	625	4	4	2	1
Subject	142000	0	155	660	4	3	2	1

The "screen" used to select comparables was:

1. Predicted price (as estimated by the regression model) within 20% of the predicted price of the subject property.
2. House area within 20% of the subject.
3. House age within 5 years of the subject's age.
4. Lot area within 35% of the subject.
5. No more than one more or fewer bedrooms.
6. No more than one more or fewer bathrooms.
7. Same year of sale.
8. Similar location coordinates in the suburb, indicating location not more than about one km from the subject.

If the comparable "passed" seven of the 8 tests, it was included in the sample as "close enough" in hedonic characteristics space to be classified as a "comparable sale." This method is certainly rather arbitrary, but common sense says it is not too different from the way people shop for houses. Certainly one could argue that this set of houses will be more similar to the subject with respect to omitted variables than would be the case for a more diverse set of houses.

Adjustments through a price differences model

Then the coefficients estimated from the regression model were used to calculate price differences, leading to an adjustment grid (table A2). This was for convenience and might be modified by further model testing, as these “plug in” estimates are actually not expected to be correct in this small sample of six sales. A “wrong sign” of the BATHTOT variable is not troublesome as this reflects effects “ceteris paribus” with house area held constant. It means that in a house with a given size and number of bedrooms, squeezing in another bath apparently squeezes out something people value more highly. Too many baths is bad design.

Table A2 Adjustment Grid

Differences	SALE1	yrdum	AREA_HSE	LAND_ARE	HSEAGE	BEDS	BATHTOT	row30d
1		0	10	130	1	-1	-1	0
2		0	0	40	0	-1	-1	0
3		0	22	112	0	0	-1	0
4		0	-10	126	0	-1	-1	0
5		0	26	23	1	-1	-1	0
6		0	-17	35	0	-1	0	0
Regrcoeff		11423	584	49	-1513	813	-1842	5222
	SALE1	yrdum	AREA_HSE	LAND_ARE	HSEAGE	BEDS	BATHTOT	locdum
1		0	5836	6327	-1513	-813	1842	0
2		0	0	1947	0	-813	1842	0
3		0	12839	5451	0	0	1842	0
4		0	-5836	6133	0	-813	1842	0
5		0	15173	1119	-1513	-813	1842	0
6		0	-9921	1703	0	-813	0	0

Summing the adjustments and adding them to the comparable properties’ prices gives us a set of “indicated values” for the subject property (Table A3).

Table A3 Indicated values and errors

	Sumadj	Valest	Errors	%error	A/S
1	11680	142680	-680	0.50%	100.50%
2	2976	131976	10024	7.10%	92.90%
3	20133	148133	-6133	4.30%	104.30%
4	1326	119326	22674	16.00%	84.00%
5	15809	145809	-3809	2.70%	102.70%
6	-9031	150969	-8969	6.30%	106.30%

Average	139815	2185	1.50%	98.50%
Std dev	11992			

In valuation practice, we would not be able to calculate these errors (the last three columns of Table A3) because the price would be unknown. However we would be able to observe the variation in "indicated prices" shown in the "Valest" column.

The fact that we have produced six different estimates of the subject property price by adjusting each of six comparable sales means that we have some evidence regarding errors in the valuation.

The data set is relatively uniform so it turned out that two schemes recommended by Colwell, Cannaday & Wu to give more weight to comparables with smaller summed absolute (or total squared) adjustments were not helpful in reducing prediction errors in the valuations (See also Whipple, 1995 p. 287-288). So in Table A3 the six indicated values were simply averaged to produce a value estimate for the subject property.

Averaging gives us a fairly precise estimate, only "off" in this instance, by 1.5%. The standard deviation of the sampling distribution of the mean (with n=6) will be 11992/2.24=5360 (2.24 is the square root of 5, or n-1). This provides an indication that a 95% confidence interval for the estimate may be on the order of $140,000 +/-10,000.

As a further check on this price estimation model, each of the comparables was valued by the same method, using the five remaining comparable sales. This gave a total of 30 valuations.

A/S is the ratio of the appraised value (following the notation of the IAAO text) to the sale price. In the thirty valuations A/S ranged from 77% to 126%. The coefficient of dispersion calculated from these valuations of the comparables was 10.2%, a measure of average deviation from the median A/S ratio. About a third of the valuations had errors of less than 5% while 13% of the valuations had errors over 20% (Table A4).

Table A4 Cumulative percentage absolute prediction errors

Errors less than	Cumulative %
5%	32.26%
10%	51.61%
20%	87.10%
30%	100.00%

While these kinds of calculations may seem tedious and overly time consuming—and they are the first time—they improve the valuation product by allowing the valuer to make representations supported by evidence regarding the accuracy of the value estimate provided. To clients interested in risk and investment returns, these estimates of the second moment of the possible price distribution would be valuable information.

The time required to produce these error analyses could be made quite small by setting up templates and Excel macros or other kinds of “do loop” programs to execute the repeated valuations quickly. Automating a spreadsheet with macros would allow a valuer to do everything reported in this appendix and more in a few minutes.

Moreover, further automation could allow valuers to test empirically the validity of their price adjustment models. Different coefficients could be used to see which would minimise prediction errors. Optimum sample sizes could also be explored.

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