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A COMPARISON BETWEEN DIVIDEND DISCOUNT MODEL AND CYCLICAL DIVIDEND DISCOUNT MODEL FOR INCOME PRODUCING PROPERTY APPRAISAL

By

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Abstract

The work is focused on the comparison between the well-known Dividend Discount Model (Gordon Shapiro, 1956) and its modifications (Two Stages Model, H-model) and the Cyclical Dividend Discount Model recently proposed by the author (d'Amato,2001) . In the last model, the value of an income producing property is more concerned about the market cycles. The work is organized as follow: The first paragraph will be concerned about the comparison between the DD model and CDD model. A presentation of the model will be also offered. In the second paragraph will be considered some differences between the two models in order to define the relationship between them and their consequences in term of property value. Final remarks will be offered at the end.

INTRODUCTION

The Dividend Discount Model methodology was developed[1] by two researchers (Gordon and Shapiro,1956) for the valuation of a perpetuity, which grows or decreases at a constant rate g. The rate g is also defined g-factor or growth factor, even if could be both negative and positive[2]. The general formula is indicated below:

$$V = \frac{D}{Y - g}$$

Where D is the dividend of the financial asset. The application of the model to income producing properties allows D to be replaced by Net Operate Income. The *Y* is the discount rate and the *g* factor is the increasing or decreasing constant percentage of property asset taken into account. Several further contributions developed and analysed the model. Among the others, Estep and Hanson and Simonotti defined the link between DD model and inflation. Nevertheless, the role of market cycle in the DD model is not evident. In fact, the perpetuity can only grow or decrease in a constant way. For this reason, according to the emerging importance of market cycles[3] other models as the "Two Stages" and H model were developed taking into account the different phases of market cycles. In addition to these models, this work show a comparison between a further model called CDD and the original Dividend Discount Model.

1. CDD model and DD model

Starting from a classification of market cycles[4] a new model was proposed[5]. In this methodology, the value of an income producing property was calculated as a sum of several distinct differences between the value at the beginning and at the end of each cyclical phase. In particular, two distinct g-factors are taken into account instead of one. The former g-factor is related to Mueller Laposa's recovery recession phase and will be defined as "g_{rr} – *g factor recovery recession*".

It can be considered as the annual average decrease of the property asset value and income in this particular real estate market phase.
The latter g-factor called will be defined as in the previous work $+ g_{ec}$ or *“g-factor expansion-contraction*”. It is the estimation of the annual increase of property asset value and its income along expansion-contraction phase. In order to simplify the relationship between the property appraisal and market cycles , only these two phases will be taken into account. The first assumption of the model is that both Recovery Recession and Expansion Contraction have similar temporal length. These lags will be defined as t_{rr} and t_{ec}.
Consequently, the sum between t_{rr} and t_{ec} can be defined the “period” of the cycle. Assuming a t_{rr} phase of recession the value of an income producing property in this interval will be

$$V_{1stPhaseRR} = \frac{NOI}{Y+g_{RR}} - \frac{NOI}{Y+g_{RR}}\frac{1}{(1+Y)^{trr}}$$

It is possible to consider a second phase after the first and the result will be a sum of the two values. In this case, the value of the asset is calculated summing the first and the second phase, as follows:

$$V_{1stPhaseRR+2ndphaseEC} = \frac{NOI}{Y+g_{RR}} - \frac{NOI}{Y+g_{RR}}\frac{1}{(1+Y)^{trr}} + $$
$$+ \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^{tec+trr}} - \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^{tec+2trr}}$$

Now can be considered a number n of phases in which
$t_{rr} = t_{ec} = n$. Therefore it will be:

$$V = \frac{NOI}{Y+g_{RR}} - \frac{NOI}{Y+g_{RR}}\frac{1}{(1+Y)^{n}} +$$
$$+ \frac{NOI}{ks-g_{EC}}\frac{1}{(1+Y)^{n}} - \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^{2n}} +$$
$$+ \frac{NOI}{Y+g_{RR}}\frac{1}{(1+Y)^{2n}} - \frac{NOI}{Y+g_{RR}}\frac{1}{(1+Y)^{3n}}\dots$$

Then:

$$V = \frac{NOI}{Y+g_{RR}}\left(1 - \frac{1}{(1+Y)} + \frac{1}{(1+Y)^{n}} + \frac{1}{(1+Y)^{2n}} + \dots\right) +$$
$$+ \frac{NOI}{Y-g_{EC}}\left(\frac{1}{(1+Y)^{n}} + \frac{1}{(1+Y)^{2n}} + \frac{1}{(1+Y)^{3n}} + \dots\right)$$

This formula can be expressed in the following way

$$V = \frac{NOI}{Y+g_{RR}}\left(1 - \frac{1}{(1+Y)^{n}} + \frac{1}{(1+Y)^{2n}} - \frac{1}{(1+Y)^{3n}} + \dots\right) +$$
$$+ \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^{n}}\left(1 - \frac{1}{(1+Y)^{n}} + \frac{1}{(1+Y)^{2n}} - \frac{1}{(1+Y)^{3n}}\dots\right)$$

Then

$$V = \left[\frac{NOI}{Y+g_{RR}} + \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]\left(1-\frac{1}{(1+Y)^n}+\frac{1}{(1+Y)^{2n}}-\frac{1}{(1+Y)^{3n}}...\right)$$

The second part of the formula is an infinitive geometric progression, whose rate is the following number

$$r = -\frac{1}{(1+Y)^n}$$

When the addition rate of an infinitive geometric progression is inside the following interval -1< r < 1 the progression will tend to the following formula:

$$\sum_{i=1}^{oo} r_i = \frac{1}{1-r}$$

The value of a property will be

$$V = \frac{1}{1+\frac{1}{(1+Y)^n}}\left[\frac{NOI}{Y+g_{RR}} + \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

and finally

$$V = \frac{(1+Y)^n}{(1+Y)^n+1}\left[\frac{NOI}{Y+g_{RR}} + \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

It is well known that DD model is based on the following assumption: Y>g. In a similar way, the CDD model has some assumptions . It is easy to understand that Y must be greater than g_{EC} and t_{RR} must be equal to t_{EC}. In fact, if t_{RR} is not equal to t_{EC} then it will not be possible to define a geometric progression.

2. A comparison between CDD and DD models.

The CDD model characteristics will be analysed through a comparison with the well known DD model. The first difference is the Overall Capitalization Rate.
Starting from the CDD model another definition of the Overall Capitalization Rate is possible. While, starting from the DD model the Overall Cap Rate R is determined as follow:

$$V = \frac{NOI}{Y-g}$$

$$V(Y-g) = NOI$$

$$\frac{NOI}{V} = (Y-g) \Rightarrow R = Y-g$$

The CDD model allows a different determination of Overall Capitalization Rate. Then

$$V = \frac{(1+Y)^n}{(1+Y)^n+1}\left[\frac{NOI}{Y+g_{RR}} + \frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

$$V = NOI\,\frac{(1+Y)^n}{(1+Y)^n+1}\left[\frac{1}{Y+g_{RR}} + \frac{1}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

$$\frac{V}{NOI} = \frac{(1+Y)^n}{(1+Y)^n+1}\left[\frac{1}{Y+g_{RR}} + \frac{1}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

Then

$$R = \frac{NOI}{V} = \frac{(1+Y)^n+1}{(1+Y)^n}\left[\frac{(Y+g_{RR})(Y-g_{EC})(1+Y)^n}{(Y-g_{EC})(1+Y)^n+(Y+g_{RR})}\right]$$

Finally

$$R = \frac{NOI}{V} = \left[(1+Y)^n+1\right]\left[\frac{(Y+g_{RR})(Y-g_{EC})}{(Y-g_{EC})(1+Y)^n+(Y+g_{RR})}\right]$$

The result is another Overall Capitalization Rate. This Cap Rate is formed by two parts. The first part is dependent on the discount rate and by the cycle (n) only, while the second part will be dependent on all the terms of the model.
Another difference between DD model and CDD one is inside the role of inflation.
In order to take into account the role of inflation the DD model was modified. In fact, Estep and Hanson[6] extended the traditional DD model trying to incorporate the differential effect of inflation[7] on the dividend growth. Because of the inflation, the DD model requires two different modifications. The former is linked to Fisher's relationship:

$$(1+Y) = (1+r)(1+if)$$

In this formula, *Y* is the discount rate, *r* is the real rate of return and *if* the expected rate of inflation. The second modification is depending on the DD model growth, or Net Operate Income growth (in the case of a real property). In fact, it can be considered divided in two parts. The first is due essentially to the real growth while the second is due to the presence of inflation, then:

$$(1+g) = (1+rg)(1+ftc)\ ,$$

Where *g* is the growth factor. This factor is composed by a real growth represented by *rg* and a flow through coefficient *ftc.* This coefficient represents the fraction of inflation, which flows to the profit. The final formula in the case of DD model will be:

$$V=\frac{NOI_0(1+rg)(1+ftc)}{[(1+r)(1+if)-1]-[(1+rg)(1+ftc)-1]}.$$

In the case of CDD model the original formula recalled below

$$V=\frac{(1+Y)^n}{(1+Y)^n+1}\left[\frac{NOI}{Y+g_{RR}}+\frac{NOI}{Y-g_{EC}}\frac{1}{(1+Y)^n}\right]$$

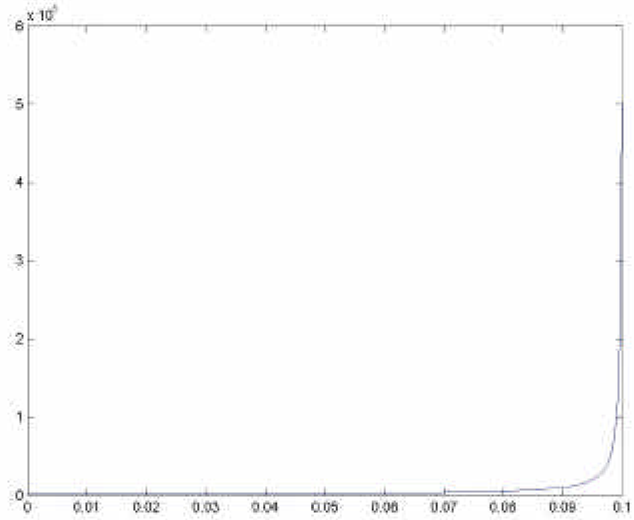
will be transformed as follow:

$$V=\frac{\left[1+(1+r)(1+if)-1\right]^n}{\left[1+(1+r)(1+if)-1\right]^n+1}\left[\begin{array}{l}\frac{NOI}{\left[(1+r)(1+if)-1\right]+\left[(1+rg_{RR})(1+ftc_{RR})-1\right]}+\\ +\frac{NOI}{\left[(1+r)(1+if)-1\right]-\left[(1+rg_{EC})(1+ftc_{EC})-1\right]}\frac{1}{\left[1+(1+r)(1+if)-1\right]^n}\end{array}\right]$$

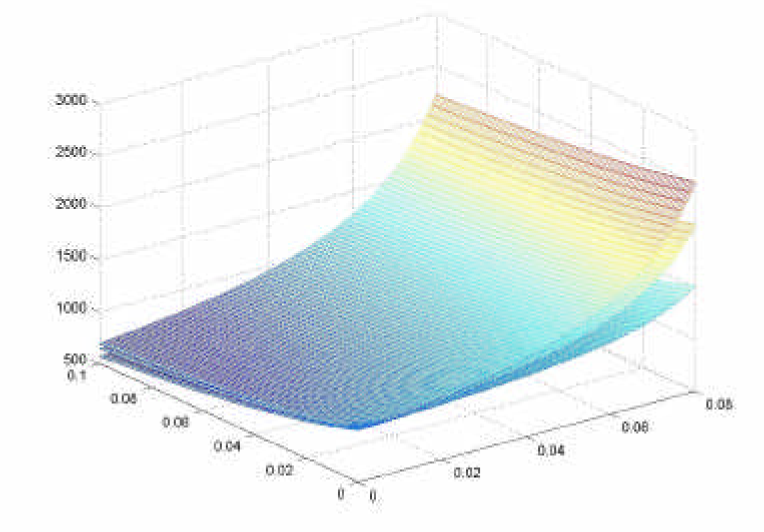
Then

$$V=\frac{\left[(1+r)(1+if)\right]^n}{\left[(1+r)(1+if)\right]^n+1}\left[\begin{array}{l}\frac{NOI}{\left[(1+r)(1+if)-1\right]+\left[(1+rg_{RR})(1+ftc_{RR})-1\right]}+\\ +\frac{NOI}{\left[(1+r)(1+if)\right]-\left[(1+rg_{EC})(1+ftc_{EC})\right]}\frac{1}{\left[(1+r)(1+if)\right]^n}\end{array}\right]$$

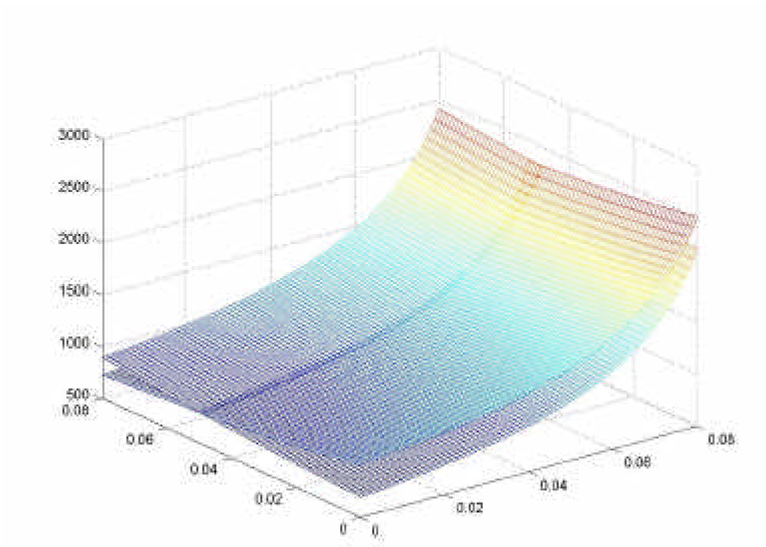
As one can see, the formula above can be defined as a Cyclical Inflation Through Model. The application of this model is not easy. The term Y or discount rate is supposed to be constant. Therefore also *if* and *r* are considered constant. This could be unreliable in a model with more than one market phase. Two terms as rg and ftc are supposed to change according to the two market phases considered (recovery recession and expansion contraction).
DD model can give further information on g-factor using the inverse formula.
It seems clear that starting from a huge amount of property knowing the value , the Net Operate Income , the Discount Rate an analysis of the $\mathbf{g_{EC}}$, $\mathbf{g_{RR}}$ and **n** is possible.
In a previous work the relationships between n and g-factors inside the CDD model and between g factor and the value have been analysed[8]. In this work , the same example will be taken into account. In fact, considering Y = .1 ; NOI equal to 100 and a variation of g factor between the values 0.01 and 0.08 the relationship between the g factor and value is indicated in the graphic 1 below:

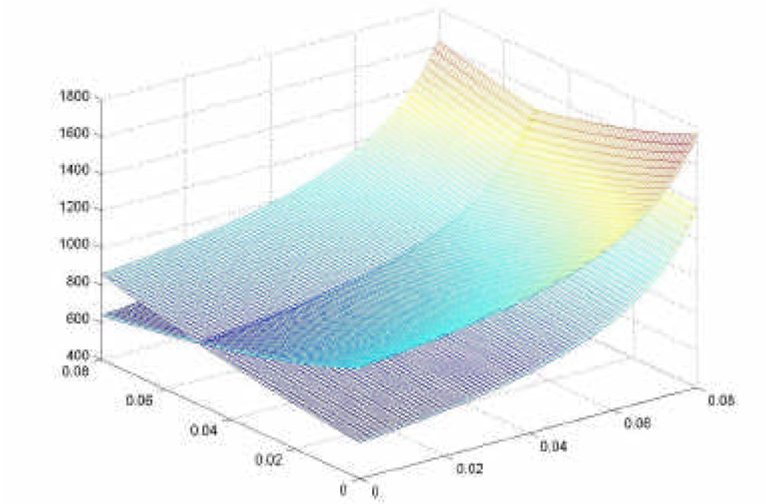


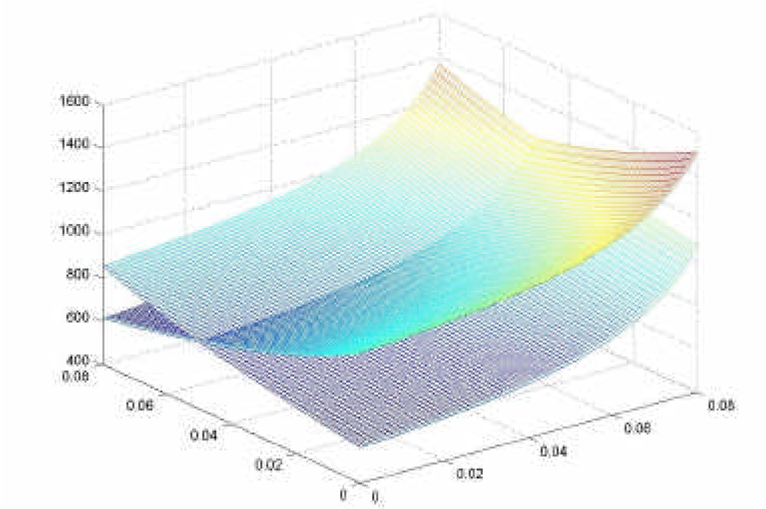
As one can see the relationship is bidimensional and observes how the property value varies with the g factor. Using the CDD model the relationship between property value and g factors has more than one dimension. The relationship will be depicted in two different conditions. In the first case the same example will be considered assuming a proportional growth between g_r and g_{ec}. In this case assuming Y=.1 a length of the cycle of 5, 15 and 20 years, a proportional growth of the two gfactors between .01 and .08 The value will be described as the surface in the graphic 2 below



If the growth of g_{RR} and g_{EC} is similar and constant then the value will assume some values describing the surface in the graphic 2. If the g_{RR} grows and the g_{EC} decrease then the property will assume different values . In the three graphics 3,4,5 below there are two different surfaces described by the property value.







The first consider a constant and equal growth of both the g_{RR} and g_{EC} between .01 and .08 at a 5, 15 and 20 years of market cycle length. In the second it is supposed that the g_{RR}, will decrease while the g_{EC} will increase. The two graphics will develop the differences between the two markets condition at a different length of market cycles.

FINAL REMARKS AND FUTURE DIRECTIONS OF RESEARCH

At the end of this paper it is possible to highlight the following considerations:

- In those markets where there are enough information, appraiser can define the property value taking into account the market cycles length and two g-factors depending on the different phases.
- The CDD model should be tested empirically
- This model allows to create a stronger relationship between the works on market cycles and the property appraisal.
- Using the inverse formula hypothesis on the g-factors can be made

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