The Faustmann Redevelopment Cycle

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Abstract

It must have been a very long time since man first discovered that buildings too rise and fall.

A forest is planted, harvested, and replanted; a machine built, used and dismantled; and likewise, a site is constructed, occupied, and reconstructed…

Perhaps the first systematic analysis of building life cycles was on costs. Surveyors and building economists in the UK first wrote on life cycle costs in the eighties. Yet these analyses focus only on the 'cost' side of the problem. Whereas on the 'benefits' side, the fact is that rent too tends to decrease, as a building depreciates. There always comes a time when rents are too low, and running costs too high, to maintain. And when such time comes, redevelopment is desirable.

This paper tries to apply the Faustmann Condition, which originally analysed the optimal rotation of a forest, to real estate redevelopment cycles. The theoretical solution for optimal time basically equates the marginal benefits and costs in delaying the decision to redevelop. This paper also discusses the potential empirical applications of this theoretical analysis using the concept of a cost-to-rent ratio.

Keywords: *Faustmann Condition, optimal reconstruction time, marginal benefits and costs of waiting, cost-to-rent ratio*

Introduction

The economic problem of rotation was first analysed in forestry. Samuelson (1976) popularised the Faustmann (1849) solution, and spelt out in no unclear terms that Irving Fisher's (1906, 1907, and 1930) one-cycle standard timing solution was wrong.

The difference between the two solutions is basically that Fisher maximised the net present value of one cycle; while Faustmann, successive cycles to perpetuity.

Scogie and Kennedy (1996) argued that British agriculturist William Marshall (1790) specified the condition correctly some 60 years ahead of Faustmann, the German forester.

By now, after a debate over two centuries, the Faustmann solution has clearly won the battle.

Perhaps Donald Soupe (1970) was the first to apply Fisher's solution to real estate. He proposed that the optimal time to develop a vacant suburban site be obtained by equating interest rate to the rate of change of the site value (with respect to time) divided by its current value – a standard solution identical to Fisher's one cycle optimal.

Of course, one may argue that the life of a building is long enough to discard successive cycles. Yet this assumption could be applicable to renovations or temporary or relatively short-lived buildings. Moreover, the long and one-cycled Fisherian solution could be regarded as a very special case of Faustmann's general solution. Moreover, the more important point is, applying the now classic Faustmann Solution could provide hypotheses capable of being tested in the empirical world, what the Fisherian one is incapable of.

Surveyors and building economists in the UK first wrote on life cycle costs in the eighties (Norman and Flanagan (1989)). Yet these analyses focus only on the 'cost' side of the problem. Whereas on the 'benefits' side, the fact is that rent too tends to decrease, as a building depreciates. There always comes a time when rents are too low, and running costs too high, to maintain. And when such time comes, redevelopment, or at lease renovation, is desirable. In this respect, the Faustmann condition for the optimisation of gains over successive cycles fits well to the problem.

Wong and Norman (1994) first used the Faustmann condition to analyse the optimal timing of renovating a mall. One of the major findings was that other things equal, renovation cycles are longer as the ratio of renovation cost to initial yearly rent increases. This provided a solid foundation for the empirical research of redevelopment and renovation cycles.

This paper outline the main features of these alternative solutions, discuss their complications and empirical applicability, and propose directions of future research in redevelopment cycles.

The Fisherian approach in real estate

Shoupe's Fisherian approach to the optimal time to develop a vacant site can be expressed as follows:

If the gross development value *V(t)* of proposed development of a function of time *t*, and the respective construction cost up to completion is *C(t),* then the site value upon completion is:

$$L(t) = [V(t) - C(t)] \tag{1}$$

The objective function of the developer is, therefore, to maximise the net present value, *L(t),* of the site. That is,

$$\underset{t}{Max}\, L(t)e^{-rt} = [V(t) - C(t)]e^{-rt} \tag{2}$$

The first order condition for maximisation is therefore,

$$[V'(t)-C'(t)]e^{-rt}-re^{-rt}[V(t)-C(t)]=0 \tag{3}$$

Or, combining this to (1) and simplifying,

$$\frac{L'(t)}{L(t)}=r \tag{4}$$

There are two main problems with this Fisherian solution.

Firstly, the assumption on the knowledge of future prices and costs and interest rates are unclear. If these information are known and expected, development would be viable as long as the net present value of the expected site is positive. And given such viability, the site would have already been developed and occupied. If not, it might as well be vacant even when the time of Shoupe's optimal development is due.

So Shoupe's solution applies only when the rental gain, net of maintenance and depreciation costs, were zero or negative before the optimal development time, but mysteriously turn positive then after. This sudden change should be justified explicitly, if one was to assume expected knowledge of market conditions.

The crux of the matter is that Fisher's forestry solution always analyses a positive value site. Should expected net gains be negative, timbre will not be planted in the first place. Rental gains on a viable site, before Shoupe's due time for optimal development, is ignored. As a result, the solution is logically inconsistent. Whereas in forestry, planting earlier on a viable site always pays, as all profits of successive cycles will be received earlier. Abandoning a viable forest is, therefore, inconsistent with maximisation.

Following this line of logic, Shoupe's solution is not optimal development timing, but speculative selling. Apparently Shoupe had implicitly assumed that the two always take place the same time. This might have been the approach of some real estate developers in practice due to limited construction loan period. Yet in forestry, while waiting to sell, it pays to plant as long as the gain in stumpage value per annum is larger than the sum of annual cost of maintenance and the interest on initial plantation cost. Similarly, with a small enough cost of transaction for arranging finance, it pays to build while waiting to sell as long as the yearly rental gain, net of maintenance and depreciation, is larger than the interest on construction cost. Should this condition of benefits and cost of waiting not hold, it probably wouldn't either at the time of sale, unless justified otherwise explicitly.

Apart from this problem of potential rental gain before development, the second problem of the Fisherian approach is, of course, the failure to recognise successive cycles after the first. The price of the development upon completion is mysteriously assumed to vary according to market conditions. Shoupe is unclear on whether these conditions are expected or stochastic. If stochastic, then the optimal timing solution is empirically indeterminate. Hence, unfortunately, Shoupe's Fisherian approach to real estate timing would not offer testable implications.

The Faustmann approach to redevelopment cycles

Instead of Fisher's one cycle solution, let's now assume Faustmann's successive cycles to perpetuity under known market conditions.

Suppose that a building receives net rent per annum *y(t)* at time *t*. This net rent decreases as *t* increases, because of depreciation and increasing maintenance costs.

Should the owner decides to redevelop it at time *T*, then the overall net present value of the building within one cycle would be:

$$Y(T) = \int_0^T y(t)e^{-rt}\,dt - Ce^{-rT} \tag{5}$$

where *C* is construction cost for redevelopment.

Ignoring the effects of inflation, by using the real rate of interest *r* (*r*=nominal minus inflation rate), the total net present value of the property would be:

$$V(T) = Y(T) + Y(T)e^{-rT} + Y(T)e^{-2rT} + \ldots = \frac{Y(T)}{1-e^{-rT}} \tag{6}$$

as the number of cycles tends to infinity.

Now the owner's objective function is to choose a desirable building life, *T*, to maximise the property's net present value, *V(T)*. The first order condition for this maximisation is therefore:

$$V'(T) = \frac{Y'(T)(1-e^{-rT}) - re^{-rT}Y(T)}{(1-e^{-rT})^2} = 0 \tag{7}$$

Or,

$$Y'(T)(1-e^{-rT}) = re^{-rT}Y(T) \tag{8}$$

Or,

$$Y'(T) = re^{-rT}\frac{Y(T)}{(1-e^{-rT})} = re^{-rT}V(T) \tag{9}$$

Differentiating (5),

$$Y'(T) = y(T)e^{-rT} + re^{-rT}C \tag{10}$$

Putting (10) into (9), and simplifying, we have the first order condition as:

$$y(T) + rC = rV(T) \tag{11}$$

This is the general condition for optimal redevelopment time. The intuition is that on the right hand side of equation (11), we have the marginal benefits of delaying redevelopment: the current rent receivable plus interest saving in delaying construction. On the right hand side, marginal lost due to a delay: the interest on the capitalised value of the property, i.e. the 'implicit rent' of the site.

In order to solve for the optimal redevelopment time, we need more specific information on how net rent *y(T)* depreciates, i.e. the knowledge of function *y*. Let's take the general assumption of a exponential decay.

Suppose

$$y(T) = Rp^T \tag{12}$$

where *R* is the initial net rent receivable under brand-new conditions; and *p*, the depreciation factor per annum; and *p* equal one minus the rate of depreciation (say *p*=1-5%=0.95, should the rate of depreciation be 5% per annum).

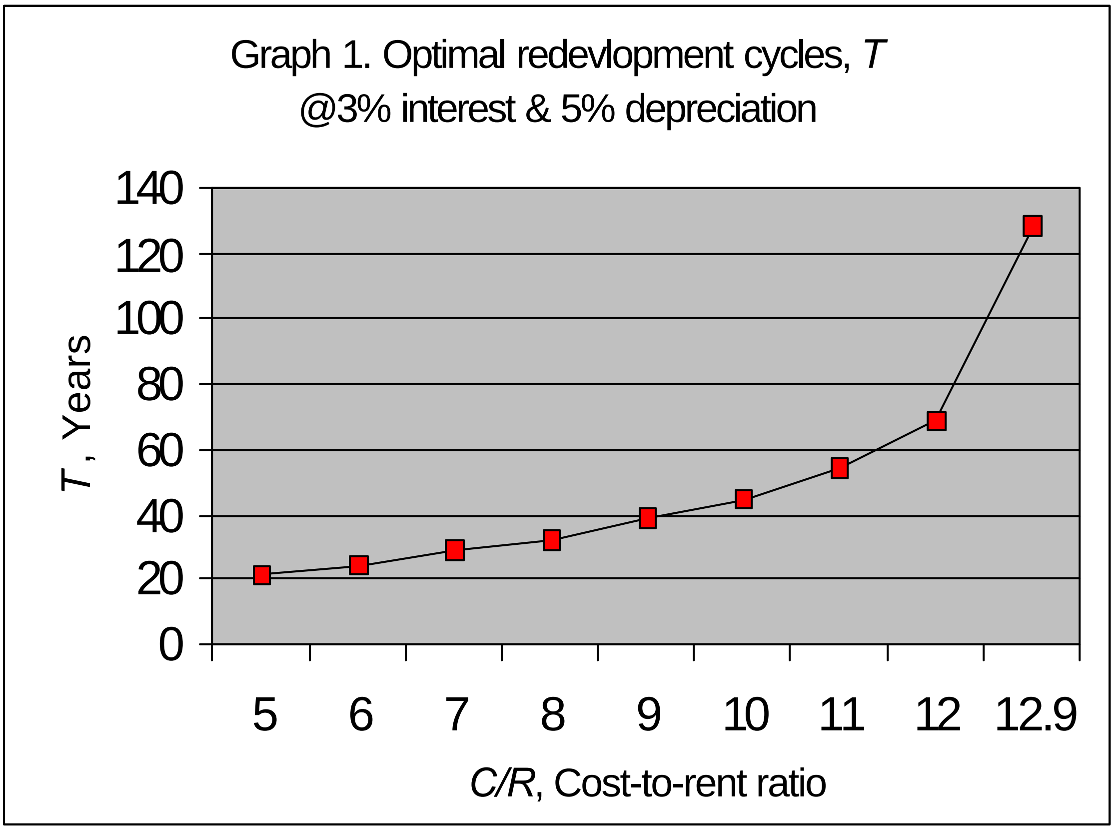
Putting (12) into the marginal condition (11), and simplifying would give the following marginal condition:

$$p^T + \frac{C}{R} - r\frac{\dfrac{1-p^T e^{-rT}}{r-\ln p} - \dfrac{C}{R}}{1-e^{-rT}} = 0 \tag{13}$$

We can see that the solution for optimal redevelopment cycle, *T*, does not depend on neither the absolute value of *C*, nor that of *R*, but their ratio *C/R*. We would therefore like to know how the solution of optimal time T behave as we vary the cost-to-rent ratio. The intuition is, of course, a higher reconstruction cost would delay redevelopment, and hence a longer cycle.

Since T cannot be expressed explicitly as a function other variables in equation (13), solving T for given ranges of the variables would give a good indication of the trend.

Suppose a site cost $10,000 per square meter to reconstruct and would receive on average $1,000 per square meter per annum upon completion. The cost to rent ratio would be $10,000/$1,000=10. And if the depreciation rate is 5% per annum and real interest rate 3%, then the optimal time obtained from equation (13) is 44.75 years. Graph 1 shows the result of repetitive calculations, using a wider range of C/R ratios. We can see, from this graph, that the optimal life, *T*, of a building is a monotonically increasing function of the cost-to-rent ratio, *C/R*. The general version of this result can be proved analytically by comparative statics (See Wong and Norman (1994)).



Empirical implications

A few testable hypotheses can be deduced from this increasing relationship between redevelopment cycle and the cost-to-rent ratio.

Firstly, given a similar construction cost, properties at urban locations, where rents are relatively high, a smaller C/R, are redeveloped more frequently. Following a similar logic, renovation cycles within these shorter redevelopment cycles, tends to be shorter too.

Secondly, as a corollary, to be compatible to shorter redevelopment and renovation cycles, leases in urban areas tends to be shorter too.

Thirdly, anchor tenants are usually offered smaller rents in the same mall. Compared to other shops renovated at similar costs per square meter, the cost-to-rent ratios for anchor-tenant shops are higher, and hence, longer leases.

Fourthly, compared to shops at similar rents in the same mall, shops requiring intensive initial investment, into renovations, equipment and otherwise, are offered longer leases – again due to a higher C/R ratio.

Finally, in addition to the empirical implications provided above, the cost-to-rent ratio analysis derived from the Faustmann Condition, can be generalised to other equipment cycles. For instance, the same equipment bringing in higher incomes may be replaced more frequently. Same model cars rented at higher priced, better-managed, car-rental companies; same model of trucks used by a profitable dairy farmer than poor farmer of corn; and even perhaps the same make of a suit worn by a highly paid CEO compared to a poor professor, are some of the examples. All these could be tested, not individually, but statistically.

Conclusion

In recent years, game theory has found its way to real estate. When interpreting lease duration, researchers now speak of moral hazards, implicit contracts and opportunistic behaviour. People may well be dishonest and opportunistic. Unfortunately, none of these notations had provided us testable implications - not even in principle.

It is not my conviction that all the hypotheses proposed in this paper are correct. Rather, to the contrary, they could all in principle be proved wrong. Yet, the important point is, the capability of being wrong is the exactly the fundamental necessary condition for a science, which great philosopher Karl Popper proposed. And that is the approach in real estate research that I am advocating.

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