Determination of the Maximum Affordable Investment Sum for Negatively Geared Properties

by

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Abstract:
Property investors can use relatively simple models to determine the investment sum, or property price, they are prepared to risk in a given institutional setting. This paper develops a framework for investing in negatively geared property in the Australian institutional setting. The robustness of the model is examined using a spreadsheet based simulation of the key parameters.

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1 Introduction

Negative gearing is a commonly used strategy employed by investors. Its attraction stems from the anticipated benefits that will accrue to those who receive moderate to high incomes, and invest in growth assets, such as shares and property. By far the most popular use of negative gearing in Australia is for residential investment property. This is due to the relatively high loan-to-value ratio (LVR), a low yield on the property and the view that this type of asset is fairly low risk. Our discussion will be restricted to residential property investment.

A negatively geared investment may be defined as an investment where the income derived from it is less than the expenses incurred from owning it during a financial year, that is, the taxable income associated with the investment is less than zero. The investor intentionally makes a loss, which may be offset against other income to reduce total tax paid by the investor. The compensation for the loss incurred by the investor during the current tax year is the expected future capital gain arising from property appreciation. The yield (or *in-going* yield) is obtained from the ratio of the rent income to the purchase price.

From an after tax perspective, a negatively geared investment may be cash flow positive, cash flow neutral or cash flow negative. A relatively high yielding investment and/or a low LVR can generate positive after tax cash flow. The choice of positive or negative after tax cash flow will reflect the investor's preferences, which may be a function of free cash flow available or risk preference.

Different types of residential investment property can provide yields in the range from four to eight percent. An investment with a yield close to eight percent can produce a positive after tax cash flow, even if it is entirely debt financed. Yields for non-residential property can range from four to twelve percent depending on the risk attached to the rental income and the potential for capital gain.

Residential properties that have the lowest return are generally older houses and apartments within a ten kilometre radius of the central business district. Properties in these locations have continued to be attractive to investors due in particular to the high capital gains consistently achieved and low vacancy rates. Developers offering new apartments to the market, which sold off the plan or at completion, have provided rental guarantees of up to seven percent for periods of one year or longer. These artificial yields are usually not sustained beyond the period of the guarantee. Yields of up to eight percent are common with managed apartments. This type of property comes under the *Managed Investments Act* (July 1998) and is regulated by the Australian Securities and Investment Commission in much the same way as managed funds.

The categories of residential investment property referred to in the preceding paragraph are not exhaustive. Indeed there are many other variations of residential property and when combined with a range of financing options, which is attracting a broad cross-section of investors, there is a greater need for analytical models to assist investors. Negative gearing has received a good deal of exposure in the popular press and in many popular financial publications. Much of this commentary takes an extreme position in either opposing or supporting this approach. Our position is more pragmatic, any prospective investment must satisfy *standard investment criteria*, which takes account of the opportunity cost of capital, if it is to be considered, regardless of its gearing. A simple example of an intertemporal investment, evaluated for a five year holding period, is provided in the appendix.

The next section introduces a relatively simple model to determine the maximum investment sum in a given environment, given the investor's funding constraint and the prevailing investment parameters. The analysis assumes that the investor has adequate equity to provide sufficient security for whatever borrowings are required. This is not unusual as investors may have other assets, such as their home or other property, that may be debt free or at least have a significant amount of equity. Before proceeding with an investment the conditions of lenders must be satisfied. Lenders view residential property as a highly attractive form of security, ranking before other types of property and also shares in blue chip companies.

Section 2 describes a relatively simple model that may be used to determine – for a given property investment environment - the maximum investment sum that may be earmarked for a property without violating the out of pocket expenses constraint that is stipulated by the investor. The term *property investment environment*, refers to a credible set of assumptions that are made about such variables as the ratio of rental income to purchase price, property holding costs as a percentage of rent, the interest rate on borrowed funds, the marginal tax rate of the investor and the magnitude of the investor's initial deposit. In this simple model, the complication of depreciation as well as its tax treatment in Australia (post July 1985) is relegated to an extended model that is described in Section 3. Section 4 analyses the sensitivity of the maximum investment sum to marginal changes in the interest rate as well as the investor's tolerable level of periodic out of pocket expenses.

2 A Simple Model for Determining the Investment Sum (No Depreciation)

Investors approach the market with knowledge of their ability to service financial commitments. Specifically the investor may or may not have a cash deposit, and may also be able to contribute a cash sum on a regular basis towards the maintenance of the investment. Market research will further reveal what types of investment properties are currently available. The analysis pursued in this paper assumes that all borrowing is of the interest only variety. Market analysis will enable the investor to collect key information for each property that will assist with the investment decision. This information includes:

- the in-going yield – the ratio of the rent to the purchase price (g)
- the operating, or holding cost (rates and property taxes, management fees, maintenance, etc), as a percentage of the rent (h)
- the interest rate on borrowed funds (i)
- the investor's marginal tax rate (t)
- the magnitude of the investor's initial deposit (D)
- the maximum annual out-of-pocket costs C that the investor is willing and able to tolerate (refer equation 1.)
- the maximum investment sum V that may be earmarked for an economically viable property without violating the ceiling placed by the investor on annual out-of-pocket expenses (refer equation 4.)

The issue of building allowance and depreciation of fixtures and fittings is deferred to the next section.

Since the investment is to be negatively geared, a loss L will be incurred. However, if the loss may be offset against the investor's other income the effective after tax cash flow is given by:

$$L - tL$$

where tL – the investor's marginal tax rate times the loss – represents a tax credit (or tax savings).

The maximum loss that may be sustained, without exceeding the investor's out-of pocket costs, C, is the difference between the total loss incurred, L, and the tax credit, tL. This must be the L that satisfies the following equality:

$$C = L - tL \qquad (1)$$

Solving for L:

$$L = \frac{C}{1-t} \qquad (2)$$

An expression for reported net taxable income P (or –L) is given by:

Net Income = Rent Income – [Operating Expenses + Interest Expense]

$$P = gV - [hgV + i(V-D)]$$

or equivalently:

$$-L = V[(g(1-h) - i] + iD \qquad (3)$$

Where gV is rental income

hgV is the holding cost (or property outgoings)

V – D is the amount of borrowed funds

i(V – D) is the interest expense

If equations (2) and (3) are summed we obtain:

$$L - L = 0 = \frac{C}{1-t} + V[(g(1-h) - i] + iD \qquad (2) + (3)$$

Equation (2) represents the *absolute value* of the loss, hence it is a positive number. Equation (3) represents the actual loss incurred, this is a negative number since the expenses by definition are greater than rental income, hence the term negative gearing. Therefore L - L = 0

By adding equations (2) and (3) one of the unknowns, L, has been eliminated. The resultant equation may then be rearranged with V a function of the known parameters.

$$V = \frac{-iD(1-t) - C}{[g(1-h) - i](1-t)} \qquad (4)$$

Example: An investor has a deposit of $40,000 and is able to contribute $400 per month (C = 12 x $400 = $4,800) to carry the investment. Market research has provided the following information:

g = 5%, h = 25%, i = 7% and t = 48.5%.

Substituting these values in equation (4) produces:

$$V = \frac{-0.07(\$40{,}000)(1-0.485)-\$4{,}800}{[0.05(1-0.25) - 0.07](1-0.485)} = \$372{,}935$$

This investor could afford an investment property with an acquisition cost of $372,935 given the prevailing scenario.[1]

[1] The acquisition cost of the investment property, represented by V, includes both the purchase price of the property and all associated purchase costs. The single largest purchase cost is stamp duty, which is levied by each State and based on a set scale.

3 Extending the Model to Incorporate Depreciation

Under current taxation rules a non-cash tax deduction is available for properties that were constructed after 18th July 1985. This is an allowance for the cost of construction, and is 4% for properties commenced between 18th July 1985 and 15th September 1987, and 2.5% for those commenced after the latter date. Using straight line depreciation, a tax deduction of 2.5% per annum of the building cost may be claimed each year for 40 years, this is commonly referred to as the *building allowance*.

Plant and equipment[2] installed in the property may be depreciated over its economic life. Different items will have a different economic life. The Australian Taxation Office (ATO) provides a schedule of depreciation rates for various items. The ATO will also accept depreciation rates provided by investors, if they are based on economic life and can be substantiated.

The building allowance and depreciation of plant and equipment are non-cash deductions since they are expenses claimed for tax purposes but are not out of pocket expenses. Due to the detail required for the inclusion of depreciation of fixtures and fittings they cannot be included in a general model of this type. However, an experienced investor can assign a suitable estimate for these deductions.

In addition to the variables introduced in the previous section, new variables are required to account for depreciation.

- building component as a proportion of property value (k)
- depreciation allowance as a percentage of the building component (d)

If the property is subject to depreciation, the after tax cash flow is given by:

$$L - tL - dkV$$

In this expression the term: dkV (building allowance) has been subtracted from $L - tL$ because the reported loss L includes dkV a non-cash expense. Depreciation of plant and equipment requires an estimate of the value of each item and its economic life. For a specific property type, an experienced investor can make a reasonable *ex-ante* guesstimate of what this might be. A straightforward approach to including depreciation of plant and equipment is to incorporate it in the building allowance. Without compromising the model, this component may assume a value of zero or the proportion prescribed for the building allowance can be increased to compensate.

As in the simpler model considered in Section 2, the expression for after tax cash flow is set to C so that we may determine the maximum loss that may be sustained without violating the investor's out-of-pocket costs constraint. The investor's on-

[2] For residential property the term *fixtures and fittings* may be more appropriate.

going contribution, C, is the residual after the tax credit and depreciation allowance have been subtracted from the loss, that is,

$$C = L - tL - dkV$$

which upon re-arrangement yields:

$$L = \frac{C + dkV}{1 - t} \tag{5}$$

In the extended model, reported net income P (= -L) includes a deduction for depreciation (namely the item dkV) so that we may write:

Net Income = Rent Income – [Operating Expenses + Interest Expense + Depreciation]

$$P = gV - [hgV + i(V\text{-}D) + dkV]$$

or equivalently:

$$-L = gV - [hgV + i(V\text{-}D) + dkV] \tag{6}$$

which may be further re-arranged as:

$$-L = V[g(1\text{-}h) - i - dk] + iD \tag{7}$$

Equations (5) and (7) are a set of simultaneous equations in the two unknowns L and V. All other variables are determined in advanced by the *informed* investor. Combining equations (6) and (7) eliminates L, leaving only one unknown, V.

$$\frac{C + dkV}{1 - t} + V[g(1 - h) - i - dk] + iD = 0 \qquad (5) + (7) = (8)$$

Let x = g(1 – h) – i – dk, then (8) may be more simply expressed as follows:

$$\frac{C + dkV}{1 - t} + xV + iD = 0 \tag{8$'$}$$

Solving for V produces,

$$V = \frac{-iD(1-t) - C}{dk + x(1-t)}$$

Substituting for x = g(1 – h) – i – dk and rearranging yields,

$$V = \frac{-iD(1-t) - C}{tdk + [g(1-h) - i](1-t)} \tag{9}$$

Equation (9) may now be employed to determine the value of an investment for a given scenario. Using the previous example and assuming that the building allowance (depreciation) is 2.5% which is applied to 50% of the value of the property (the building component) the value of V may then be obtained from (9).

Example: An investor has a deposit of $40,000 and is able to contribute $400 per month ($4,800 annually) to carry the investment.
Market research has provided the following information:

g = 5%, h = 25%, i = 7% and t = 48.5%, d = 2.5%, k = 50%.

$$V = \frac{-0.07(\$40{,}000)(1-0.485)-\$4{,}800}{0.485(0.025)(0.5)+[0.05(1-0.25)-0.07](1-0.485)}$$

V = $584,731

For a property that does not qualify for the building depreciation allowance, the maximum value this investor could afford to invest was $372,935. This result was obtained using the model described by equation (4) previously. Equation (9) will produce the same result if the depreciation parameter, d, is set to zero.

The magnitude of the difference ($584,731 - $372,935 = $211,796) for an investment property that is subject to the depreciation allowance and a property constructed prior to July 1985 is considerable. The investor's out-of-pocket contribution, C, is unchanged. The explanation for the substantial increase in the value of the investment is due to the introduction of the non-cash expense, depreciation and the higher tax credit that it generates.

The absolute value of the loss using equation (5) is:

$$L = \frac{C + dkV}{1-t} = \frac{\$4{,}800 + 0.025(0.5)(\$584{,}731)}{1-0.485} = \$23{,}513$$

The components of the loss are:

Tax credit	tL = (0.485)($23,513)	=	$11,404
Depreciation	dkV = (0.025)(0.5) ($584,731)	=	$7,309
Investor's contribution		=	$4,800

An income statement for each of the properties considered is provided in the appendix. The code for an Excel user defined function, written in VBA (Visual Basic for Applications), is also provided.

4 Sensitivity of the Model to Interest Rates and Investor's Contribution

The sensitivity of the investment sum V to changes in the interest rate i and/or the investor's out of pocket contribution C is considered in this section. But first a short aside on the breakdown of the model when the interest rate is less than or equal to a boundary value denoted by i*.

Observe that the numerator of equation (9), -iD(1-t) – C, is negative for all positive values of i, D, t and C. Therefore, if expression (9) is to yield a positive value for V, its denominator must also be negative. In other words, the following inequality must hold:

$$tdk + [g(1-h) - i](1-t) < 0$$

which may be rewritten[3] as:

$$i > i^* \tag{10}$$

where:

$$i^* = tdk/(1-t) + g(1-h) \tag{11}$$

As an illustration of this exercise we may use the data provided at the previous example, to evaluate i* as follows:

$$i^* = \frac{(0.485)(0.025)(0.5)}{(1-0.485)} + .05(1-.25) = 0.04927..$$

Exhibit 1 depicts the relationship between V and i over the range i = 0% to i = 12% when all remaining variables of the model are held constant. It is easily seen that all values of V corresponding to an i > i* (i < i*) are positive (negative). Finally, the value of V is undefined[4] at i = i*.

Given that the model generates inappropriate values of V when i ≤ i*, the reader is entitled to ask under what circumstances the model yields meaningful results in a real world situation. To this matter the analysis now turns.

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[3] The inequality : $tdk + [g(1-h) - i](1-t) < 0$

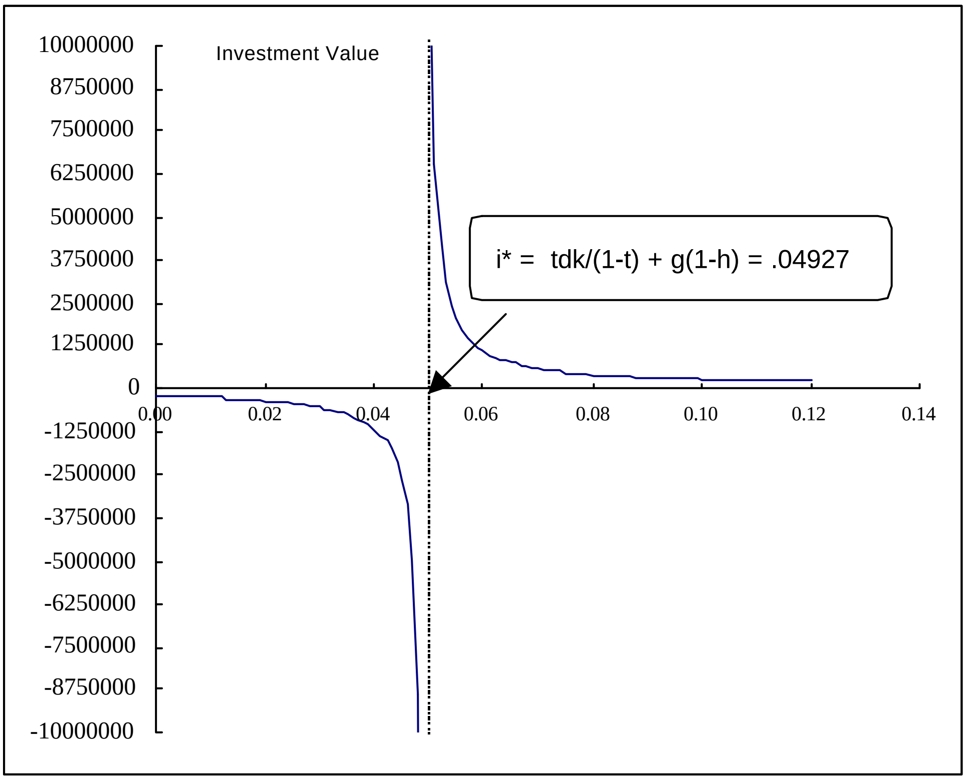
may be rearranged as : $tdk + g(1-h)(1-t) < i(1-t)$

Dividing both sides by (1-t), the following result is obtained:

$i > tdk/(1-t) + g(1-h)$ or $i > i^*$ where $i^* = tdk/(1-t) + g(1-h)$

[4] It is also true that the first derivative of V with respect to i evaluated at i = i* is undefined. This has implications for obtaining an elasticity measure of V with respect to i. Such a measure would be useful if the investor wanted to determine the percentage change in the investment sum arising from a 1% change in the interest rate.

Exhibit 1



Substitution of i* as defined in (11) into inequality condition (10) permits the latter to be rewritten as:

$$i > tdk/(1-t) + g(1-h) \qquad (10)'$$

The depreciation component in (10)′, tdk/(1-t), depends on the marginal tax rate, which in the preceding example was set at the maximum of 48.5%. As the marginal tax rate decreases, with g and h held constant, the right hand side of (10)′, will also decrease. While the marginal tax rate may be lower for some investors, it is more likely that it will be close to the maximum allowed as additional income (positive or negative) is subject to the investor's highest marginal rate.

In addition, the depreciation parameter, d, has been set to its minimum value of 2.5%, which refers only to the building allowance. Depreciation of fixtures and fittings can also be substantial, particularly during the first five years if the property was new at the time of purchase. The average depreciation rate during the early years of ownership could be closer to 5%. As fixtures and fittings are typically depreciated using the declining balance method, to maximise tax deductions during the early years of the investment, the depreciation benefit is less each year.

The ideal situation for a negatively geared investor is to obtain the highest possible depreciation deduction and highest acceptable tax credit. These two factors require the parameters d and t to be as large as possible, which increases the likelihood of the denominator of equation (9) being positive and thereby invalidating the negative gearing model described by (9).

Whilst an investor may have initially disclosed an acceptable out of pocket contribution C, it is useful to examine how the acquisition cost, V, changes as C changes.

It may be shown[5] that when the model retains validity (i.e. i > i*), V increases by constant increments in response to constant increments in C, all other variables held constant. Note however, that this constant rate of change in V is larger, the lower is the interest rate i.

The mathematical results alluded to above, may be further reinforced by analysing the contents of the following sensitivity table (Table 1) that was constructed in Excel using data from the above example. Examination of this table reveals how V responds to different combinations of (i, C) when all other variables are held constant.

The explosive nature of the model as i declines toward i* (= .04927 in our example) is also illustrated in the sensitivity table. For instance, for any given C (say $4,800), V increases dramatically as i falls toward i*. In our example when interest rates are close to 5% the values for V are unusually large indicating that the model is approaching instability. For interest rates of 5.5% and above the results are quite robust.

Further inspection of this table uncovers the positive linear relationship existing between V and C. In particular, observe that when i is 10%, V experiences successive constant increments of $45,933 in response to successive $1200 increments in C. Then, when i is lower (say at i = 8%), V's successive constant increments escalate to $75,829. Finally at an interest rate of 6% the constant increase in V for each additional $1200 annual contribution is $217,195.

Table 1 indicates that when interest rates are low, a 1% change in interest rates produces a greater change in the value of the acquired investment, V, than when rates

[5] These results may be easily established. Consider expression (9) for the investment sum V. This may be rewritten more simply as:

$$V = \frac{m - C}{n} \qquad \text{where: } m = -iD(1\text{-}t) \text{ and } n = tdk + [g(1\text{-}h) - i](1\text{-}t).$$

Furthermore, if attention is confined to cases where the model retains validity (i.e. n < 0), then the rate of change in V with respect to C is constant and positive. In other words $\frac{\partial V}{\partial C} = \frac{-1}{n} > 0$ as n < 0. What happens to this constant positive rate of change if the interest rate were at a lower level ? If n remains negative, the lower i pushes n closer to zero. This implies that $\frac{\partial V}{\partial C}$ - although still constant and positive – will be greater the closer n is to zero.

are high. Consider for instance, the change in V that arises when i moves from 6% to 7% as compared to moving from 11% to 12%. As indicated immediately below, the resultant change in V is much higher when the initial interest rate is lower.

	Initial V	Final V	Change in V
Increase in i from 6% to 7%	$1,092,489	$584,731	$507,758
Increase in i from 11% to 12%	$225,931	$199,643	$26,288

Table 1: Sensitivity of V to Changes in i and C

	$0	$1,200	$2,400	**$3,600**	**$4,800**	$6,000	$7,200	$8,400	$9,600
5.0%	$2,746,667	$5,946,667	$9,146,667	$12,346,667	**$15,546,667**	$18,746,667	$21,946,667	$25,146,667	$28,346,667
5.5%	$384,068	$790,847	$1,197,627	$1,604,407	**$2,011,186**	$2,417,966	$2,824,746	$3,231,525	$3,638,305
6.0%	**$223,710**	**$440,905**	**$658,100**	**$875,294**	**$1,092,489**	$1,309,683	$1,526,878	$1,744,072	$1,961,267
6.5%	$165,309	$313,457	$461,605	$609,753	**$757,901**	$906,049	$1,054,198	$1,202,346	$1,350,494
7.0%	$135,082	$247,494	$359,906	$472,319	**$584,731**	$697,143	$809,555	$921,967	$1,034,379
7.5%	$116,604	$207,170	$297,736	$388,302	**$478,868**	$569,434	$660,000	$750,566	$841,132
8.0%	$104,139	$179,968	$255,798	$331,627	**$407,457**	$483,286	$559,115	$634,945	$710,774
8.5%	$95,163	$160,380	$225,598	$290,815	**$356,033**	$421,250	$486,467	$551,685	$616,902
9.0%	$88,391	$145,602	$202,813	$260,024	**$317,235**	$374,446	$431,657	$488,868	$546,079
9.5%	$83,100	$134,055	$185,011	$235,966	**$286,921**	$337,877	$388,832	$439,788	$490,743
10.0%	$78,852	$124,785	$170,718	$216,651	**$262,584**	$308,517	$354,450	$400,383	$446,316
10.5%	$75,366	$117,178	$158,990	$200,801	**$242,613**	$284,425	$326,237	$368,049	$409,861
11.0%	**$72,454**	**$110,823**	**$149,193**	**$187,562**	**$225,931**	$264,301	$302,670	$341,039	$379,408
11.5%	$69,985	$105,436	$140,886	$176,337	**$211,787**	$247,238	$282,688	$318,139	$353,589
12.0%	**$67,865**	**$100,810**	**$133,754**	**$166,699**	**$199,643**	$232,588	$265,532	$298,476	$331,421

For all i > i* it is also possible to arrive at some elasticity measures for V with respect to i and C. The respective formulae for these elasticities[6] are given by:

Elasticity of V with respect to C $= \mathring{a}_C = \frac{\partial V}{\partial C} \times \frac{C}{V} = \frac{C}{iD(1-t)+C}$ (12)

and

Elasticity of V with respect to i $= \mathring{a}_i = \frac{\partial V}{\partial i} \times \frac{i}{V} = \frac{i(1-t)\{-D[tdk+g(1-h)(1-t)]-C\}}{\{tdk+[g(1-h)-i](1-t)\}\{-iD(1-t)-C\}}$ (13)

[6] An explanation of the derivation of ε_I is provided in the appendix.

Table 2: Elasticity of V with respect to C - evaluated for selected values of i and C

	$10	$1,200	$2,400	$3,600	**$4,800**	$6,000	$7,200	$8,400	$9,600
5.0%	0.0096	0.5381	0.6997	0.7775	0.8233	0.8535	0.8748	0.8908	0.9031
5.5%	0.0087	0.5144	0.6793	0.7606	0.8090	0.8412	0.8640	0.8811	0.8944
6.0%	0.0080	0.4926	0.6601	0.7444	0.7952	0.8292	0.8535	0.8717	0.8859
6.5%	0.0074	0.4726	0.6419	0.7289	0.7819	0.8176	0.8432	0.8625	0.8776
7.0%	0.0069	0.4542	0.6247	0.7140	**0.7690**	0.8062	0.8331	0.8535	0.8694
7.5%	0.0064	0.4372	0.6084	0.6997	0.7565	0.7952	0.8233	0.8446	0.8614
8.0%	0.0060	0.4213	0.5929	0.6860	0.7444	0.7845	0.8137	0.8360	0.8535
8.5%	0.0057	0.4066	0.5782	0.6728	0.7327	0.7741	0.8044	0.8275	0.8457
9.0%	0.0054	0.3929	0.5642	0.6601	0.7214	0.7639	0.7952	0.8192	0.8381
9.5%	0.0051	0.3801	0.5508	0.6478	0.7104	0.7541	0.7863	0.8110	0.8307
10.0%	0.0048	0.3681	0.5381	0.6360	0.6997	0.7444	0.7775	0.8031	0.8233
10.5%	0.0046	0.3568	0.5260	0.6247	0.6894	0.7350	0.7690	0.7952	0.8161
11.0%	0.0044	0.3462	0.5144	0.6137	0.6793	0.7259	0.7606	0.7875	0.8090
11.5%	0.0042	0.3362	0.5033	0.6031	0.6695	0.7169	0.7524	0.7800	0.8021
12.0%	0.0040	0.3268	0.4926	0.5929	0.6601	0.7082	0.7444	0.7726	0.7952

Table 3: Elasticity of V with respect to i - evaluated for selected values of i and C.

	$10	$1,200	$2,400	$3,600	**$4,800**	$6,000	$7,200	$8,400	$9,600
5.0%	-67.676	-68.205	-68.366	-68.444	-68.490	-68.520	-68.542	-68.557	-68.570
5.5%	-8.610	-9.116	-9.281	-9.362	-9.411	-9.443	-9.466	-9.483	-9.496
6.0%	-4.601	-5.085	-5.253	-5.337	**-5.388**	-5.422	-5.446	-5.464	-5.479
6.5%	-3.140	-3.605	-3.775	-3.862	-3.915	-3.950	-3.976	-3.995	-4.010
7.0%	-2.384	-2.831	-3.002	-3.091	**-3.146**	-3.183	-3.210	-3.231	-3.246
7.5%	-1.922	-2.352	-2.523	-2.615	-2.672	-2.710	-2.738	-2.760	-2.776
8.0%	-1.610	-2.025	-2.196	-2.289	-2.348	-2.388	-2.417	-2.439	-2.457
8.5%	-1.385	-1.786	-1.957	-2.052	-2.112	-2.153	-2.183	-2.207	-2.225
9.0%	-1.215	-1.603	-1.774	-1.870	-1.931	-1.974	-2.005	-2.029	-2.048
9.5%	-1.083	-1.458	-1.628	-1.725	-1.788	-1.832	-1.864	-1.889	-1.908
10.0%	-0.976	-1.339	-1.509	-1.607	-1.671	-1.716	-1.749	-1.774	-1.795
10.5%	-0.889	-1.241	-1.410	-1.509	-1.574	-1.619	-1.653	-1.679	-1.700
11.0%	-0.816	-1.158	-1.326	-1.425	**-1.491**	-1.537	-1.572	-1.599	-1.620
11.5%	-0.754	-1.086	-1.253	-1.353	-1.419	-1.467	-1.502	-1.530	-1.552
12.0%	-0.701	-1.023	-1.189	-1.290	-1.357	-1.405	-1.441	-1.469	-1.492

Using the same parameter values that were used in the example provided above and calculating the elasticities for C = $4,800 and i = 7%, they are:

$$\varepsilon_{C=4800} = 0.769 \quad \text{and} \quad \varepsilon_{i=7\%} = -3.146$$

In other words, a 1% increase in the amount that the investor is willing to be out of pocket will increase the amount of the affordable investment sum by 0.769 of 1% all other parameter values held constant. In much the same way, one may say that a 1% increase in the interest rate necessitates a reduction of 3.146% in the affordable investment sum if the investor's out of pocket expenses are to remain unchanged (as well as all other model parameters).

Further inspection of expression (12) confirms that ε_C is positively related to C with an upward bounded value of unity[7]. Moreover, for a given level of C, ε_C will be lower the higher are i and D and the lower is t. In other words, the sensitivity of the investment sum to a 1% change in C is greater the higher are i and D and the lower is t.

Unfortunately, the complexity of expression (12) does not permit a straightforward unambiguous explanation of the behaviour of ε_i. Some insight into this complexity may be gained from Exhibit 1.

In this exhibit consider the curvilinear relationship between V and i to the right of i*. Next, along this curve, consider the behaviour of the ratio i/V and the modulus of the gradient $|\delta v/\delta i|$ as i is increased beyond i*. The behaviour of these determine the behaviour ε_i whose formula is redrafted as:

$$e_i = -\left|\frac{\partial V}{\partial i}\right| \frac{i}{V} \qquad (13)'$$

As i is increased, it is observed that $|\delta v/\delta i|$ falls at the same time as the ratio i/V rises. Which of these opposing effects predominate will obviously determine the behaviour of ε_i. Ultimately the behaviour of ε_i is an empirical matter that will depend on the remaining parameters of the model. For those parameter values that have been employed to produce the relationship depicted in Exhibit 1, it may be shown that ε_i becomes a successively larger negative number (i.e. closer to zero) the higher is the interest rate. For instance, when the interest rate is 6% (all other parameters held constant), ε_i = -5.388 and at 11%, ε_i = -1.491 (refer table 3). What this means is that the higher is the interest rate, the less responsive is the affordable investment sum to a 1% change in the interest rate.

These last results are in sympathy with much less precise measures of responsiveness that may be determined indirectly from the entries of Table 1. Illustrative computations are reproduced in Table 4 for the case of an investor whose annual out of pocket contribution is $4800.

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[7] If both the numerator and denominator of expression (11) are divided by C, one obtains the following alternative formula for ε_C: $e_C = \left[\frac{iD(1-t)}{C} + 1\right]^{-1}$

Inspection of this alternative form, reveals that ε_C gets progressively closer to unity as C rises, all other parameter values held constant.

Table 4: Rough Measures of the Responsiveness of V to a 1% increase in i

Move in i	% Move in i	Move in V	% Move in V	$\frac{\text{\% change in V}}{\text{\% change in i}}$ *
6% to 7%	16.67%	$1,092,489 to $584,731	-46.48%	-2.79
11% to 12%	9.09%	$225,931 to $199,643	-11.64%	-1.28

* Entries in this column may be taken as very rough measures of the elasticity of V with respect to i. Strictly speaking better measures of elasticity would be given by arc elasticities. These are given by -3.94 and -1.42 respectively (refer to the appendix for the calculation of these arc elasticities).

If i moves from 6% to 7% - a percentage change of 16.67% - V drops from $1,092,489 to $584,731 – a percentage fall of 46.48% (see Tables 1 and 4). A crude measure of responsiveness may now be taken as the ratio of the latter percentage change to the former. This yields –2.79. If the same approach is applied to computing elasticity when i rises from 11% to 12%, the approximate elasticity measure now moves to –1.28. This as expected, is closer to zero.

Interest rates have a major impact on negatively geared investment decisions. The current low interest rate environment encourages risk taking. If rates increase significantly, the fortunes of some heavily geared investors on variable rates could alter dramatically.

Table 5: Impact of Higher Interest Rates When V = $584,731

	Loss	Interest Expense	Tax Credit	Depreciation	Investor's Contribution
5.0%	($12,618)	($27,237)	$6,120	$7,309	$811
5.5%	($15,342)	($29,960)	$7,441	$7,309	($592)
6.0%	($18,066)	($32,684)	$8,762	$7,309	($1,995)
6.5%	($20,789)	($35,408)	$10,083	$7,309	($3,397)
7.0%	**($23,513)**	**($38,131)**	**$11,404**	**$7,309**	**($4,800)**
7.5%	($26,237)	($40,855)	$12,725	$7,309	($6,203)
8.0%	($28,960)	($43,578)	$14,046	$7,309	($7,605)
8.5%	($31,684)	($46,302)	$15,367	$7,309	($9,008)
9.0%	($34,408)	($49,026)	$16,688	$7,309	($10,411)
9.5%	($37,131)	($51,749)	$18,009	$7,309	($11,813)
10.0%	($39,855)	($54,473)	$19,330	$7,309	($13,216)
10.5%	($42,578)	($57,197)	$20,651	$7,309	($14,619)
11.0%	($45,302)	($59,920)	$21,972	$7,309	($16,021)
11.5%	($48,026)	($62,644)	$23,293	$7,309	($17,424)
12.0%	($50,749)	($65,368)	$24,613	$7,309	($18,827)
12.5%	($53,473)	($68,091)	$25,934	$7,309	($20,230)
13.0%	($56,197)	($70,815)	$27,255	$7,309	($21,632)
13.5%	($58,920)	($73,539)	$28,576	$7,309	($23,035)
14.0%	($61,644)	($76,262)	$29,897	$7,309	($24,438)
14.5%	($64,368)	($78,986)	$31,218	$7,309	($25,840)
15.0%	($67,091)	($81,710)	$32,539	$7,309	($27,243)

It will be observed from Table 5 that as the interest rate increases, the investor is required to contribute additional out-of-pocket funds to maintain the investment. For each half percent increase the annual additional contribution is $1,403.

The interest cost is deductible, which increases the tax credit flowing to the investor. This has a moderating effect on the additional out-of-pocket contribution. However, the additional exposure to risk, if rates are not fixed for the holding period, directly impacts on the opportunity cost of capital.

An even greater concern for the investor is the security of cash flow. As interest rates increase the economy as a whole is affected. If there is a downturn in economic activity, vacancy rates may rise and rents will be more difficult sustain. The riskiness of the investment will increase and the new more risky environment will cause the opportunity cost of capital to rise, making the investment less attractive.

5 Conclusion

The use of negative gearing as an investment strategy is frequently employed by investors purchasing residential property. The benefits accruing to such investors are the result of taxation benefits, arising from debt servicing and depreciation, and ability of the investment to generate future capital gains. A model was developed to determine the maximum affordable investment sum in the Australian environment. The model was then used to examine sensitivities arising from changes in interest rates and the annual out-of-pocket contribution made by the investor.

The results show that when the interest rate is very low, close to a specific lower bound, the investment sum is highly sensitive to interest rate changes. As the interest rate increases, the absolute value of interest elasticity becomes progressively smaller. The model also shows that, for a given interest rate, constant increases in the investor's annual out-of-pocket costs are associated with constant increases in the investment sum. These increases are greater when the interest rate is lower.

Finally, it was shown that if the interest rate increases from a very low level, which is evident in the current economic climate, by a significant amount, the annual out-of-pocket cost could increase substantially if rates were not fixed. However, locking in interest rates will merely control expenses. Higher interest rates could have a negative impact of rental income and change the risk profile of the investment.

Appendix

Multi-Period Investment Return

After determining the amount to invest the next step for the investor is to measure the performance of the investment. This will require some assumptions about the future economic climate and the holding period for the investment. During the holding period many of the initial parameters can reasonably be assumed to remain unchanged.

Rent as a proportion of value will remain the same, however, rent income will increase due to capital gain. Holding costs, depreciation and interest rates remain constant. The building allowance is a constant 2.5% for 40 years and the building component is a fixed sum that is known in advance. Depreciation of fixtures and fittings has been set to zero for tractability of the model, the effect of this is to understate the benefits conferred on the investor. An interest-only loan for a fixed period of time, the holding period, may be assumed and the investor's marginal tax rate is unchanged.

The return to the investor may be determined from the present value of after tax cash flows during the holding period and the present value of the reversion, this may be obtained from equation (A1).

$$NPV = CF_0 + \frac{CF_1}{(1+r)} + \frac{CF_2}{(1+r)^2} + ... + \frac{CF_n}{(1+r)^n} \qquad \text{(A1)}$$

where CF_j represent after tax cash flows (positive or negative) in year j = 1, 2,…, n. The holding period is n years and CF_n represents all cash flows in period n, including the reversion.

The discount rate, r, represents the investor's opportunity cost of capital for this type of investment. If the NPV is positive the investment meets the criteria for proceeding.

A key assumption at this point is rate of growth in capital appreciation during the holding period. Due to the cyclical nature of the property market, which when coupled with the fickleness of the economy, a good estimate for growth may be difficult to obtain. A somewhat less challenging question may be to consider what capital growth rate is necessary to achieve an NPV of zero and then consider whether this rate is achievable.

The example previously presented is used to illustrate the investor's position for a holding period of 5 years. The initial value of the investment is $584,731 and all other parameters are unchanged. The opportunity cost of capital for assuming this level of risk is presumed to be 12%.

Table A1: Calculation of Investor's Return for Holding period of Five Years

Year	0	1	2	3	4	5
Investment Value (V_t)		**$584,731**	**$596,625**	**$608,762**	**$621,145**	**$633,781**
Rent Income (5% of V_t)		$29,237	$29,831	$30,438	$31,057	$31,689
Expenses						
Operating costs (25% of rent)		($7,309)	($7,458)	($7,610)	($7,764)	($7,922)
Interest expense (7% of loan)		($38,131)	($38,131)	($38,131)	($38,131)	($38,131)
Depreciation allowance		($7,309)	($7,309)	($7,309)	($7,309)	($7,309)
Taxable Income (Loss)		**($23,513)**	**($23,067)**	**($22,612)**	**($22,147)**	**($21,674)**
Tax credit		$11,404	$11,187	$10,967	$10,741	$10,512
Depreciation allowance		$7,309	$7,309	$7,309	$7,309	$7,309
Investor's contribution		**($4,800)**	**($4,570)**	**($4,336)**	**($4,097)**	**($3,853)**
After tax cash flows						
Rent Income		$29,237	$29,831	$30,438	$31,057	$31,689
Tax credit		$11,404	$11,187	$10,967	$10,741	$10,512
Interest expense		($38,131)	($38,131)	($38,131)	($38,131)	($38,131)
Investor's contribution	($40,000)	($4,800)	($4,570)	($4,336)	($4,097)	($3,853)
Sale price (end year 5)						$646,673
Loan outstanding						($544,731)
Capital Gains Tax(on 50% of gain)						($23,883)
After Tax Cash Flows	**($40,000)**	**($2,291)**	**($1,683)**	**($1,062)**	**($429)**	**$78,276**
Present Values (r = 12%)	**($40,000)**	**($2,045)**	**($1,341)**	**($756)**	**($273)**	**$44,416**
NPV	**$0**					

Calculation of capital gains tax		
Sale price		$646,673
Purchase price	$584,731	
Less accumulated depreciation	($36,546)	$548,185
Capital Gain		$98,488
CGT (on 50% of gain @ 48.5%)		**($23,883)**

During the holding period of 5 years, an average annual growth rate of 2.0342% in V will produce an NPV of zero when the opportunity cost of capital is set at 12%. The out-of pocket costs decrease each year since the ratio of deductible expenses to rent income declines. Depreciation represented the building allowance only. For newly constructed properties the deduction for depreciation would be considerably higher. Hence, the after tax cash flows are somewhat conservative, that is, an average growth rate of less than 2% per annum would be required to achieve an NPV of zero. The investor may now determine if this growth rate, or better, is achievable during the next five years.

Property Constructed prior to July 1985 (building depreciation allowance is zero).

Price (acquisition cost = V)		$372,935
Deposit		$40,000
Loan		$332,935
Rent Income (5% of V)		**$18,647**
Expenses	Operating costs (25% of rent)	($4,662)
	Interest expense (7% of loan)	($23,305)
Taxable Income (Loss)		**($9,320)**
Tax credit (48.5% of Loss)		**$4,520**
Investor's contribution (Loss + tax credit)		**($4,800)**

Property Constructed after September 1987 (building depreciation allowance is 2.5%).

Price (acquisition cost = V)		$584,731
Deposit		$40,000
Loan		$544,731
Rent Income (5% of V)		**$29,237**
Expenses	Operating costs (25% of rent)	($7,309)
	Interest expense (7% of loan)	($38,131)
	Depreciation expense	($7,309)
Taxable Income (Loss)		**($23,513)**
Tax credit (48.5% of Loss)		**$11,404**
Depreciation reversed		**$7,309**
Investor's contribution		**($4,800)**

```
Function Neg_Gearing(Rent, Hold, Interest, MTR, Depn, BC, Deposit, Contribution)
   ' ======= Variables required =========
   ' Rent = rent as proportion of V (V = the investment sum)
   ' Hold = holding costs (property outgoings)as % or rent
   ' Intr = interest rate on borrowed funds
   ' Deposit = deposit paid by investor
   ' Con = annual contribution by investor
   ' MTR = marginal tax rate
   ' Depn = depreciation allowance
   ' BC = building component (proportion of V subject to depn)
   ' Neg_Gearing = V = the value of the investment (property acquisition cost)

   V1 = -Interest * Deposit * (1 - MTR) - Contribution
   V2 = MTR * Depn * BC + (Rent * (1 - Hold) - Interest) * (1 - MTR)
   Neg_Gearing = V1 / V2

End Function
```

Elasticity of V with respect to i

Derivation of formula for point elasticity

$$V = \frac{-iD(1-t) - C}{tdk + [g(1-h) - i](1-t)}$$

$$e_i = \frac{dV}{di}\frac{i}{V}$$

$$\frac{dV}{di} = \frac{-D(1-t)[tdk + [g(1-h) - i](1-t)] - (-iD(1-t) - C)[-(1-t)]}{[tdk + [g(1-h) - i](1-t)]^2}$$

$$\frac{i}{V} = \left[\frac{tdk + [g(1-h) - i](1-t)}{-iD(1-t) - C}\right] i$$

$$e_i = \frac{i[-D(1-t)[tdk + [g(1-h) - i](1-t)] + (1-t)[-iD(1-t) - C]]}{[tdk + [g(1-h) - i](1-t)][-iD(1-t) - C]}$$

$$e_i = \frac{i(1-t)[-D[tdk + g(1-h)(1-t)] - C]}{[tdk + [g(1-h) - i](1-t)][-iD(1-t) - C]}$$

Calculation of the Arc Elasticity when C = $4,800

$$e_i^{Arc} = \frac{(V_2 - V_1)}{(i_2 - i_1)}\frac{(i_1 + i_2)/2}{(V_1 + V_2)/2}$$

Change in i from 6% to 7%:

$$e_i = \frac{(584731 - 1092489)}{7\% - 6\%}\frac{(6\% + 7\%)/2}{(1092489 + 584731)/2} = -3.9356$$

Change in i from 11% to 12%:

$$e_i = \frac{(199643 - 225931)}{12\% - 11\%}\frac{(11\% + 12\%)/2}{(225931 + 199643)/2} = -1.4207$$