A CONSUMPTION BASED EXPLANATION OF EQUITY AND HOUSING PROPERTY RETURNS

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ABSTRACT

We apply the consumption-based asset pricing model (CCAPM) to the returns on Finnish housing property and equity investments. We assume that the probability distribution is jointly log-normal and estimate the representative consumer’s first order conditions for an optimal consumption and portfolio choice by maximum likelihood. We allow the consumer’s preferences to exhibit smooth transition with respect to a recession state variable. While the CCAPM does not fit the equity returns and does not explain the ‘equity premium puzzle’ present in stock returns, it provides a satisfactory description to the time varying returns on housing property investments.

Keywords: Housing property investment, asset pricing, CCAPM, stochastic discount factor

INTRODUCTION

Markets for housing property investments are subject to dramatic price changes. These changes are usually as hard to explain as are price changes in the equity and bond markets. Unlike equity and bond prices, real estate price series are hard to construct due to the heterogeneous nature of real estate and the lack of central market places. Furthermore, the actual rent earned by housing property can not often be observed, because a large part of housing property is owner occupied and thus does not pass through rental markets. For example in Finland, about 64 percent of the housing property is owner occupied. The owner occupant of housing property earns an implicit rent in the form of housing services, on which there is no market valuation. Housing property is, however, an alternative to e.g. equities and bonds in a consumer’s portfolio. Thus the general pricing relations should apply to housing investments as well as to assets with well-defined daily market valuations.

The development of consumption-based asset pricing theory is one of the major advances in financial and macro economics during the last 25 years. In the theory of finance the expected excess return on any risky asset over the risk-free return is explained by the quantity of risk times the price of risk. In CAPM the excess return on the market portfolio measures the price of risk, while the beta captures the quantity of risk. In a standard consumption-based asset pricing model (CCAPM) of the type studied by Rubinstein (1976), Lucas (1978), Grossman and Shiller (1981) and Hansen and Singleton (1983), the quantity of market risk is, instead, measured by the covariance of the excess return with consumption growth, while the Arrow-Pratt coefficient of relative risk aversion of a representative consumer equals the price of risk.

Two commonly known empirical facts raise questions in macroeconomics and finance. Firstly, why is the average real stock return so high in relation to the average short-term real interest rate? This is the “equity premium puzzle” of Mehra and Prescott (1985). Secondly, why is the volatility of real stock returns so high in relation to the volatility of the short-term interest rate? This is the “volatility puzzle”, attributed to Campbell (2001). We examine, if these well known puzzles are present in housing market returns by applying CCAPM, which is one of the basic tools in the theory of modern macro economics and financial asset pricing. All current asset pricing models are derived from the consumption-based model. Thus, one cannot expect that asset pricing models such as the CAPM hold, but consumption-based models do not.
Case, Quigley and Shiller (2001) have recently calculated wealth effects caused by both equities and housing property on consumption. They found that the wealth effect of housing property was both statistically significant and twice as large as the effect of the stock market. Like stock markets, also housing markets experience bull and bear markets, but prices tend to be less volatile than they are for shares, with more moderate peaks and troughs than the stock markets. A rise in house prices is more likely to be seen as a permanent gain in wealth by a homeowner than a rise in share prices. Furthermore, homeowners can borrow against housing equity. Cash extracted from housing in this way appears to have fuelled consumption directly in the U.S., as other sources of income growth and wealth have fallen away. The findings of Case, Quigley and Shiller (2001) are in accordance with our results in this paper. For a rational forward looking representative consumer or investor there is a closer link between consumption and housing property returns than between consumption and stock market returns. Thus, the CCAPM is expected to give a better explanation to housing returns than the traditional capital asset pricing model (CAPM).

Although the consumption-based asset pricing model should apply to all assets it has been tested empirically mainly on stocks. In fact the validity of the CCAPM has not been formally tested on real estate\(^1\) even though the CCAPM has a somewhat appealing intuition for application to real estate. Firstly, the CAPM assumes a myopic behaviour of investors, who optimise with regard to the outcomes of their present decisions on the portfolio value at the next day only. The CCAPM instead, describes the intertemporal choice problem of a representative consumer, who optimises the expectation of a time separable utility function and uses financial assets to transfer one’s wealth between different periods and states of the world.

Secondly, while the economic intuition behind the traditional wealth-based CAPM is portfolio theory, the economic intuition behind the consumption CAPM is somewhat different. The CCAPM is based on the declining marginal utility of wealth as consumption increases. It is an investor’s consumption that matters most fundamentally, as individual utility is defined directly on consumption. The advantage of this approach is that a unit of consumption good provides a common denominator valid at all dates. Wealth is only a means to the end of consumption. The basic idea is that an asset that, ceteris paribus, yields a higher return when investors are worse off, i.e. when consumption is down, anyway and less when they are better off, i.e. when consumption is up, anyway will reduce the risk in their welfare. That kind of asset acts as a kind of hedge against the changes in the level of consumption.

According to the consumption-based asset pricing model risk corrections to asset prices should be driven by the covariance of asset payoffs with consumption. The consumption-based model focuses on the fundamental desire of more consumption rather than in intermediate objectives such as mean and variance of portfolio returns. This nature of consumption-based CAPM might give at least a partial explanation\(^2\) to the often perceived abnormally high Sharpe-ratio of real estate. According to Geltner (1989) the real estate consumption betas appear to be much higher than the real estate

\(^1\) At least we have not seen any such research.

\(^2\) An important explanation is often of course the appraisal smoothing.
stock market betas, at least after correction for smoothing. This suggests that the expected return risk premium that would be predicted by the CCAPM would be higher than that predicted by the traditional CAPM.

We apply the CCAPM to Finnish housing property and equity return data by assuming a jointly conditional log-normal distribution of the variables. The bivariate model is estimated by maximum likelihood. Instead of assuming constant preferences, we allow the subjective discount factor and the degree of risk aversion to follow a smooth transition process about a transition point of the state variable. The approach allows us to consider smooth regimes or states of the world, such that consumers and investors get more risk averse during ‘bad times’ and less risk averse during ‘good times’. This kind of behaviour may well explain a fraction of the famous ‘equity premium puzzle’.

The paper proceeds by introducing first the basic consumption-based CAPM in section II. Section III first considers a habit formation model in consumption, a theoretical framework, which can at least partly explain the ‘equity premium puzzle’. Then we suggest a smooth transition function to be included in the basic model, which may capture the habit formation behaviour in consumption. Section IV displays the data and the results of the estimation. Finally, section V concludes the results.

CONSUMPTION-BASED ASSET PRICING MODEL

Consider a typical consumer with a flow of income. He or she faces two kinds of problems. Firstly, the consumer has to decide how to allocate present and future consumption among goods and services. Consumption decisions are also savings decisions from which the funds available for investment arise. This is known as the consumption-savings problem of the consumer. Secondly, one has to decide how to invest among various assets. The demand for an asset is determined by the consumers’ desire for smooth fluctuations in consumption over time. This is called the portfolio selection problem. The two problems are interrelated, since the consumer can affect his or her consumption path by transferring wealth between different time periods through portfolio selection. Both of the problems involve making decisions under uncertainty, considering simultaneously the probabilistic notions of expected return and risk.

The basic pricing equation comes from the first-order condition, which is an Euler equation, for the consumption decision. The basic model is based on an infinite period utility maximization problem, which can be solved by dynamic optimisation. Consider, however, the intertemporal choice problem of an investor who can trade freely in some asset i at a price \( p_i \) and can obtain a payoff \( x_{t+1} \) on the asset held from time \( t \) to time \( t+1 \). The asset is any kind of security, which transfers wealth from period \( t \) to period \( t+1 \). To name a few, the asset can be a one-period zero-coupon bond with price \( p_t \) and payoff equal to 1, a “risk-free” rate with price 1 and payoff \( 1+R' \) or a common stock or housing property with price \( p_t \) and payoff \( x_{t+1} \). For example, if an investor buys a common stock today, the payoff next period is the asset price \( (p_{t+1}) \) plus dividend \( (d_{t+1}) \). We obtain the first-order condition, the Euler equation, for an optimal consumption and portfolio choice,
\[ p_t u'(c_t) = E_t [\beta u'(c_{t+1}) x_{t+1}] . \]

The left hand side of (1) is the loss of utility if the investor buys another unit of the asset; the right hand side is the expected discounted increase in utility the investor obtains from consuming the extra payoff at \( t+1 \). That is, the investor equates marginal cost and marginal benefit. To obtain equilibrium, the consumer buys more or less of the asset until the first-order condition holds. The direct utility function \( u \) is increasing, reflecting a desire for more consumption, and concave, reflecting the declining marginal value of additional consumption.

The subjective discount factor \( \beta \) captures the investor’s impatience. Furthermore, the curvature of the utility function generates aversion to risk and to intertemporal substitution: the investor prefers a consumption stream that is steady over time and across states of nature. Convenient power utility form is often used for utility:

\[ u(c_t) = (c_t^{1-\gamma} - 1)/(1-\gamma). \]

The limit as \( \gamma \rightarrow 1 \) is \( u(c_t) = \log_e(c_t) \), which corresponds to the risk neutral case.

While the power utility has some practical advantages, it has an undesirable property that links to important concepts. In the model the elasticity of intertemporal substitution is the reciprocal of the coefficient of relative risk aversion. Hall (1988) argues that this linkage is not appropriate because the elasticity of intertemporal substitution concerns the investor’s willingness to move consumption between different time periods. The concept is well defined even if there is no uncertainty, whereas the coefficient of relative risk aversion concerns the investor’s willingness to move consumption between different uncertain states of the world.

Equation (1) can be expressed as (3) to obtain the formula for the asset price. The formula traces the pricing of all assets into a single idea, where the price equals the expected payoff. The risk stems from the macroeconomic risks and consumption risk, underlying each asset’s value.

\[ p_t = E_t [\beta u'(c_{t+1}) / u'(c_t) x_{t+1}] . \]

Equation (3) is the central asset pricing formula. Most of the theory of asset pricing consists of specialization and manipulations of this formula. We may divide (3) by \( p_t \) to obtain

\[ 1 = E_t \left[ \beta u'(c_{t+1}) / u'(c_t) (x_{t+1} / p_t) \right] \]

\[ 1 = E_t \left[ \beta u'(c_{t+1}) / u'(c_t) ((p_{t+1} + d_{x_{t+1}}) / p_t) \right] \]

\[ 1 = E_t \left[ \beta u'(c_{t+1}) / u'(c_t) (1 + R_{t+1}) \right] , \]

where \( 1 + R_t \) is the gross return on the asset. For the power utility (2)

\[ 1 = E_t \left[ \beta (c_{t+1} / c_t)^\gamma (1 + R_{t+1}) \right] . \]

The parameter vector \( (\beta, \gamma) \) is commonly estimated by minimum distance methods, e.g. by applying the Generalized Method of Moments (GMM) estimation.
Often used way to break up the basic pricing equation (3) is to define the stochastic discount factor $m_{t+1}$:

$$m_{t+1} = \beta u'(c_{t+1}) / u'(c_t).$$

The stochastic discount factor is also often called the marginal rate of substitution after (6): it is the rate at which the investor is willing to substitute consumption at time $t+1$ for consumption at time $t$.

The basic pricing equation can now be expressed as:

$$p_t = E_t (m_{t+1} x_{t+1}).$$

If we use gross simple rate of return $(1 + R_i, t+1)$ we have:

$$1 = E_t [m_{t+1} (1 + R_i, t+1)].$$

It is helpful to write:

$$E_t [m_{t+1} (1 + R_i, t+1)] = E_t [m_{t+1}] E_t [(1 + R_i, t+1)] + \text{Cov}_t [m_{t+1}, R_i, t+1].$$

Then substituting into (8) and rearranging gives:

$$1 + E_t [1 + R_i, t+1] = \frac{1 - \text{Cov}_t [R_i, t+1, m_{t+1}]}{E_t [m_{t+1}]}.$$

Equation (8) must hold for any asset, including real estate. An asset that low high covariance with the stochastic discount factor must have a high expected simple return. Such an asset tends to have low returns when investors have high marginal utility i.e. in times when consumption is down, for example due to a slump. Thus investors demand a large risk premium to hold such an asset.

Capital asset pricing models are general equilibrium models of asset prices, the formal derivation of which always involves unrealistic simplifying assumptions. CCAPM is no exception. The CCAPM is usually viewed as being more general than the traditional CAPM because it allows a multi-period or continuous time world. But the CCAPM still requires such typical assumptions as complete markets or homogeneous expectations. Furthermore, real estate has characteristics, which are not associated with other assets that break the standard assumptions of capital asset pricing models, such as the CAPM. These characteristics include illiquidity, long holding period, big unit size and high transaction costs. However, the CCAPM has a somewhat appealing intuition for application to real estate, especially compared to a number of other asset pricing models.

**NEW MODELS**

The consumption-based asset pricing models have often proved disappointing empirically because the CCAPM has often indicated implausibly high levels of risk aversion. The so called ‘equity premium puzzle’ is a result of this. He and Modest (1995) suggest that one important reason for the disappointing performance of the CCAPM is
the presence of market frictions. The most recent explanation, however, is the ‘habit formation’ of consumption\(^3\).

Habit formation can explain why consumers’ reported sense of well-being often seems to be more related to recent changes in consumption rather than to the absolute level of consumption. Equilibrium versions of price models which take habit formation into account assume that a representative agent has a momentary utility function that depends on both the current rate of consumption and weighted index of past consumption rates. Higher rates of consumption in the recent past imply, ceteris paribus, lower level of utility from a given current consumption rate. That is, current consumption decisions have an effect on the entire path of future consumption through the habit index.

Campbell and Cochrane (1999) specify that people slowly develop habits for higher and lower consumption. They specify an external, or ‘keep up with the Joneses’ form of habit formation, developed by Abel (1990). In the model the ‘habits’ form the ‘trend’ in consumption. Campbell and Cochrane replace the utility function \(u(c_t)\) with \(u(c_t - x_t)\), where \(x_t\) denotes the level of habits:

\[
(11) \quad u_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \left( \frac{(c_{t+j} - x_{t+j})^{1-\gamma} - 1}{1-\gamma} \right) \right],
\]

where \(x_t\) is treated as an external habit. In this model the agent’s risk aversion varies with the level of consumption relative to the habit. It is convenient to work with the surplus consumption ratio \(S_t\), a state variable, defined by

\[
(12) \quad S_t = \frac{c_t - x_t}{c_t},
\]

which gives the fraction of total consumption that is surplus to subsistence or habit requirements.

While theoretically appealing, the model has two inconveniences for empirical applications. Firstly, the habit variable is not observed and it has to be estimated or one has to use a proxy variable instead. Secondly, in the habit model consumption must always be above habit for utility to be well-defined. We consider a model where preferences evolve through a smooth transition between two extreme regimes. Instead of assuming that an economy just has two discrete states, expansion and contraction, say, it may be more convenient and realistic to assume a continuum of states between the two extremes. As the second argument, we may expect that agents may not all act promptly and uniformly at the same moment. There may be varying delays in agents’ reactions. To apply the idea, we specify a model in which the subjective discount factor and risk aversion depend on consumption growth or the growth of wealth relative to some threshold value.

Let us consider a composite consumption good $c_t$ with price $q_t$ at date $t$. At this date the representative consumer possesses an external income $y_t$ and a portfolio of assets, whose allocation vector $\alpha_{t-1}$ has been decided at a previous date. This endowment allows the agent to consume a quantity $q_t$ and to update the portfolio to a new allocation vector $\alpha_t$. The budget constraint at time $t$ is

\begin{equation}
q_t c_t + \alpha_t P_t = y_t + \alpha_{t-1} P_t.
\end{equation}

We apply the utility function given by (2) which is optimised with respect to the future consumption and portfolio plans, and used to determine the current consumption and portfolio allocation such that the budget constraint (13) is satisfied. We obtain the Euler equation as the first-order condition for the maximum and get

\begin{equation}
1 = E_t \left[ \frac{p_{j+1,t}}{p_{j,t}} \left( \frac{q_{t+1}}{q_t} \right) \beta \left( \frac{c_{t+1}}{c_t} \right)^\gamma \right]
\end{equation}

for $j = 1, 2$ where $p_j$ is the price of equity and housing, respectively.

The power utility has the practical advantage that we may combine various rates of growth according to the following expression

\begin{equation}
1 = E_t \left[ \beta \exp(\log \frac{p_{j+1,t}}{p_{j,t}}) - \log \frac{q_{t+1}}{q_t} - \gamma \log \frac{c_{t+1}}{c_t} \right].
\end{equation}

We restrict our attention to joint conditional log-normal distribution of the growth rates in (8.5). We infer from the moment generating function of a gaussian variable $X$ the rule $E[\exp(X)] = \exp[E(X) + \frac{1}{2}V(X)]$ to obtain

\begin{equation}
1 = \beta \exp \left\{ E_t \left[ \log \frac{p_{j+1,t}}{p_{j,t}} - \log \frac{q_{t+1}}{q_t} - \gamma \log \frac{c_{t+1}}{c_t} \right] \right\} \\
+ \frac{1}{2} V_t \left[ \log \frac{p_{j+1,t}}{p_{j,t}} - \log \frac{q_{t+1}}{q_t} - \gamma E_t \log \frac{c_{t+1}}{c_t} \right].
\end{equation}

By taking logs on both sides, we have
For equity and housing property returns, j=1,2. We observe that (17) is an ARCH-M specification for the returns since the last term is \(-\frac{1}{2}\) times the variance.

By denoting 
\[
\Delta \log p_{t+1} = \log (p_{t+1}/p_t) = R_{\Delta p_{t+1}}, \quad \Delta \log q_{t+1} = \log (q_{t+1}/q_t) \quad \text{and} \quad \Delta \log c_{t+1} = \log (c_{t+1}/c_t)
\]
we allow the model to have a more general form

\[
R_{j,t+1} = \Delta q_{t+1} - \log \beta_1 + \gamma_1 \Delta c_{t+1} + \left( -\log \beta_2 + \gamma_2 \Delta c_{t+1} \right) G(\varphi, r; s_{t+1}) - \frac{1}{2} \sigma^2_{j,t},
\]

where G is a bounded continuous transition function, commonly bounded between zero and unity. It can be seen that (18) is locally linear in consumption growth and we will assume that the combined risk aversion parameter vector \((\gamma_1, \gamma_2, G)\) is a function of a transition variable \(s\). If \(G\) is bounded between 0 and 1, the combined parameter fluctuates between \(\gamma_1\) and \(\gamma_1 + \gamma_2\). Similarly, the combined subjective discount factor parameter vector \((\beta_1, \beta_2, G)\) fluctuates between \(\beta_1\) and \(\beta_1 + \beta_2\). This property makes it possible to characterize equity and property markets with dynamic properties in expansion being different from those in contraction. Granger and Teräsvirta (1993) and Teräsvirta (1988) contain further details on these smooth transition regressions (STR) models and their properties.

For the transition function \(G\) we choose the logistic smooth transition STR (LSTR) specification, defined as

\[
G(\varphi, r; s_t) = \left\{ 1 + \exp \left[ -\frac{\varphi}{\sigma_s} (s_t - r) \right] \right\}^{-1}
\]

such that \(\varphi > 0\).

The transition function (19) is a monotonically increasing function of the switching variable \(s_t\). The location parameter \(r\) determines where the transition occurs. The slope parameter \(\varphi\) indicates how rapid the transition from zero to unity is as a function of \(s_t\). If the slope parameter is ‘large’, rescaling it becomes important. We standardize the argument of \(G\) by dividing by \(\sigma_s\), the sample standard deviation of the transition variable. We define the transition variable \(s_t\) as

\[
s_t = \frac{c_t}{c_{t-4}},
\]

which defines the state as the ratio of present consumption to the consumption one year ago in quarterly data. This resembles the model proposed by Abel (1990), where the utility function is a power function of the ratio \(c_t/x_t\), where \(x_t\) is the time-varying habit or subsistence level.
DATA AND THE RESULTS

The house price, rental and running cost data used to compute the housing property returns in Finland has been received from the Finnish Statistics. Price data is based on the information gathered from major real estate agencies and is provided quarterly. However, the rental and cost figures are only reported for the second period of each year. The rental information is based on a survey among the landlords and running cost data is based on the financial statements statistics of housing corporations in Finland. We have estimated the rental and running cost figures for the first, third and fourth quarter in each year by interpolating.

Even though the real rents and costs may differ slightly from our estimates the difference is so small that it does not cause any significant problems, knowing that the total return on housing property was dominated by the price changes in the sample period, not by rental or cost changes.

The return index for Finnish housing property is computed using the following formula.

\[ I_t = I_{t-1} \times \frac{[P_t + (M_{R_t}-R_{C_t})*3]}{P_{t-1}} \]

- \( I_t \): value of the index at time \( t \)
- \( P_t \): moving quarterly arithmetic average house price / m\(^2\) in quarter \( t \)
- \( M_{R_t} \): average monthly rent / m\(^2\) in quarter \( t \)
- \( R_{C_t} \): average monthly running cost / m\(^2\) in quarter \( t \)

Quarterly returns are computed as \( \log (I_t - I_{t-1}) \).

The rest of the quarterly data is taken from Bloomberg’s OECD Main Economic Indicators Database. The series taken are gross private consumption, the consumer price index and the share price index. The sample period is from 1983:Q2 to 2001:Q1. The first two sample moments of the data are displayed in table 1.

Table 1: Statistics of nominal returns, consumption growth and change in price level (annualised)

<table>
<thead>
<tr>
<th>Series</th>
<th>Average</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return on equity</td>
<td>21.9%</td>
<td>28.4%</td>
</tr>
<tr>
<td>Return on housing property</td>
<td>8.7%</td>
<td>8.5%</td>
</tr>
<tr>
<td>Consumption growth</td>
<td>2.4%</td>
<td>2.4%</td>
</tr>
<tr>
<td>CPI change</td>
<td>3.3%</td>
<td>1.3%</td>
</tr>
</tbody>
</table>

The average annualised consumption growth has been 2.4%, while the price level has increased 3.3% on the average. Thus, the average real consumption growth has been negative during the sample period. This can be explained by an extraordinary severe slump in the early 1990’s. During the period 1990:Q1-1993:Q3 real GDP declined by 21%.
We test for the presence of the ARCH(1) property in equity and housing market returns by using the test proposed by Engle (1982). The test statistics are $\chi^2(1) = 0.446$ for equity and $\chi^2(1) = 0.532$ for housing market returns with p-values 0.50 and 0.47, respectively. Thus, the returns do not exhibit autoregressive conditional heteroscedasticity and therefore we may fix the variance terms in (18) into constants.

We estimate (18) as a bivariate model by maximum likelihood first by running a genetic algorithm and then using the Berndt, Hall, Hall and Hausman algorithm to fine-tune the solution. Table 2 displays the parameter estimates of model (18). Parameters $\sigma^2_e$ and $\sigma^2_h$ are the variances of equity and housing market returns, respectively. Covariance $\sigma_{eh}$ is not statistically different from zero.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\log \beta_1$</td>
<td>0.007</td>
<td>0.389</td>
</tr>
<tr>
<td>$-\log \beta_2$</td>
<td>-0.035</td>
<td>0.005</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>1.886</td>
<td>0.002</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>-2.137</td>
<td>0.035</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>307.216</td>
<td>0.000</td>
</tr>
<tr>
<td>$R$</td>
<td>1.021</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma^2_e$</td>
<td>0.018</td>
<td>0.000</td>
</tr>
<tr>
<td>$\sigma^2_h$</td>
<td>0.001</td>
<td>0.000</td>
</tr>
</tbody>
</table>

The first parameter is not statistically different from zero. Hence, the subjective discount factor fluctuates between 1 and 0.965 and the coefficient of relative risk aversion fluctuates between 1.886 and −0.251. The reasonable lower bound is, however zero, while the value equal to one corresponds to the risk neutral case. The value of the location parameter is 1.021 for the transition corresponding to 2.1% consumption growth, year on year, which is somewhat smaller than the average consumption growth, 2.4%.

Any model of expected returns may be viewed as a model of the stochastic discount factor. It is equivalent to a pricing kernel such that the expected product of any asset return with the stochastic discount factor equals one as given by (5). Figure 1 displays the stochastic discount factor, subjective discount factor and the coefficient of relative risk aversion. The stochastic discount factor has a mean value 0.984 implying average annual subjective discount rate of 6.7%. Just before the slump period in the early
1990s the stochastic discount factor jumps to unity. Other extreme values are observable in the early 1980s and after the first quarter of the year 2000. In this model the stochastic discount factor is also the intertemporal marginal rate of substitution, which is the discounted ratio of marginal utilities in two successive periods for the consumer.

Figure 1: Subjective discount factor, coefficient of relative risk aversion and stochastic discount factor

The coefficient of relative risk aversion is displayed in Figure 1 together with the index of consumers’ confidence. The former has the left-hand scale and the latter the right-hand scale, respectively. The risk aversion parameter governs both risk aversion and the elasticity of intertemporal substitution, which measures the willingness of the consumer to adjust planned consumption growth in response to investment opportunities. In the power utility, larger risk aversion also implies smaller elasticity of intertemporal substitution. It can be observed that the estimated consumers’ risk preference varies from risk loving to risk averse behavior. Commonly, the estimated coefficient of relative risk aversion is found to be about 1 or 2. Using the linear version of (18) by dropping the transition function G, we obtain an estimate \( \gamma = 1.267 \). Our average value for the coefficient is 0.421 in the nonlinear model. In the early 1990s the degree of risk aversion begins to grow and consumer confidence decline, such that the value of the coefficient of relative risk aversion is at the maximum during the slump. The other peak values are obtained simultaneously with the peak values of the subjective discount factor. Movements in the coefficient of relative risk aversion are captured quite well by the movements of the index of consumer confidence. Their contemporaneous correlation is –0.58.
The CCAPM implies that an asset’s expected return is greater, the smaller its covariance with the stochastic discount factor. The explanation is that an asset whose covariance with (6) is small tends to have low returns in the state where investor’s marginal utility of consumption is high. This occurs when consumption itself is low. The consumer considers such an asset risky, because it fails to deliver wealth precisely when wealth is more valuable. Therefore a large risk premium is required to hold the asset. The average returns are 21.9% and 8.7% for equity and housing property investments, respectively. The covariances with the stochastic discount factor are, however, 0.042 and 0.019 for equity and housing investment returns, respectively. This is contrary to the model’s prediction.

Figure 2 on the next page displays the predictions of the model, represented by the smoother curves, together with the observed values. The predictions of equity returns are too smooth to capture the time varying equity premium. The stochastic discount factor has a volatility of 6.3%. Hansen and Jagannathan (1991) have derived the lower bound

\[
\frac{\sigma(m)}{E(m)} \geq \left| \frac{E(R^e)}{\sigma(R^e)} \right|
\]

for the stochastic discount factor (6) consistent with the market risk premium. The right hand side is the Sharpe ratio, and if excess return \( R \) is on the efficient frontier, the Sharpe ratio is the slope of the tangent line. The average long run Sharpe ratio is in the range of 0.3-0.5. Since \( E(m) \approx 1 \), \( \sigma(m) \) should be in the range of 30-50%. Our value 6.3% is quite too small to fit the equity premium puzzle.

The model predicts an average annual equity return of 8.1%, while 21.9% is observed. The latter is, however, abnormally high due to the dominance of the Nokia stock in the Helsinki Stock Exchange. The model’s predictions have a standard deviation of 4.5%, while the observed returns have a volatility of 28.4%. This is called the volatility puzzle and it originates from the low volatility of consumption equal to 2.4%. The model has \( R^2 = 0.08 \), which is comparable to several empirical findings on fitting stock returns.

Visual inspection of Figure 2 reveals that the model provides a reasonable fit to the housing property returns. The model implies an unconditional expected return of 11.6%, while 8.7% is observed. The model’s fit has a standard deviation of 4.5%, while the observed return series has a volatility of 8.5%. The model has \( R^2 = 0.20 \) which is larger than for equity returns. Although the value is low, due to the variability of the actual observations, the model captures quite well the long-term market movements. Figure 2 displays also the implied quarterly real risk-free rate \((1+R_f^q)\), which equals the inverse of the stochastic discount factor. The implied average annual risk-free rate is 7.2% with a volatility of 6.2%.
CONCLUSIONS

In this paper we have applied the general consumption based asset pricing model to the returns on equity and housing property investment. Instead of assuming constant preferences, we have allowed both the subjective discount factor and the coefficient of relative risk aversion to have smooth transitions with respect to a state or threshold variable. Concerning equity returns, much of the equity premium puzzle still remains to be solved. The second author has been working with a utility function, which is nonseparable in consumption and the flow of liquidity services. Liquidity can be always transformed into either consumption or investments. The results are promising in this field.

The CCAPM gives a better explanation to housing property returns. This may be explained by the special characteristics of housing market investments:

(1) Both housing consumption and investments on housing property are very much consumer related,

(2) Housing property investments are dependent on the location, while equity investments are international, and there may be asymmetric and imperfect information on the characteristics of the housing property,

(3) Housing market is illiquid in the sense that the holding periods are long: households do not buy a new house every time as interesting opportunities arise.
Concerning further research, it is possible to disaggregate the housing market data into city specific markets and test the moment conditions implied by the model in these sub-markets. A natural extension is also to apply the model to more broad categories of real estate investments, instead of housing property only.

REFERENCES


