USES FOR BETAS IN PROPERTY PORTFOLIO CONSTRUCTION

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Abstract
Some fund managers use portfolio optimisation models to help them decide how much to allocate to each sector of the property market (with sectors defined typically by use and region). The models use past average periodic performance of each sector to search for combinations of the sectors that have offered the highest rate of return for a given amount of risk in the portfolio. The risk is measured by the weighted standard deviation of returns of each sector and their correlation coefficients. Usually, the average performance is based upon property indices and these models are relevant only for those funds which hold diversified property portfolios.

This paper explores the potential to use the Beta to judge the risk of including properties from one sector within a property portfolio. Because the Beta measures how volatile a sector has been, compared with the market average, it is appropriate as a measure of risk for diversified property funds. Because the Beta for the whole fund is the weighted average of the Betas of each sector, it is simple to calculate portfolio Betas and portfolio returns with different weightings to each property sector.

The paper compares the bases, critical assumptions and interpretations of the Markowitz and Sharpe approaches to portfolio optimisation and their suitability for guiding allocations to the major sectors of the property investment market. The restrictive assumptions of both models, their applications to real estate and their legitimacy in determining property sector weightings are considered.

The paper reports the results of tests applying both approaches to periodic returns for Australian property sectors. The tests demonstrate that the two approaches suggest similar sector weightings for given degrees of risk aversion, although some differences are evident at low levels of risk. The tests show that both approaches tend to reduce portfolio construction to choices between the two most favoured sectors (unless constraints are placed on the sector weightings). Further analysis of past returns indicates that Betas appear to be slightly more stable over time than standard deviations and correlation coefficients between the sectors.

The paper concludes that, despite some theoretical objections to applying the capital asset pricing model to portfolio decisions, the Beta is a measure that can help in adjusting sector weightings for property portfolios.
Uses for Betas in property portfolio construction

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Part 1: Introduction

Investors and fund managers are aware of the need for diversification and must gauge which combinations of investments will lower risk most effectively. In building a portfolio of income-producing properties, they must decide how much prominence to give to different categories of properties, often segmented by the type of buildings, their uses and by their locations. Modern portfolio theory offers measures of diversification and risk which are one (but rarely the sole) input to portfolio restructuring.

Past periodic returns of groups of properties enable objective calculations of diversification and are used on the assumption that the past covariances (or correlation coefficients) are a guide to their likely diversity in future. The most common method is mean-variance optimisation, which builds the covariance into a measure of portfolio risk. This paper considers the alternative of using Betas, similarly calculated from periodic returns from samples of properties, as a guide to the kinds of properties that can be combined to lower portfolio risk.

In Part 2 of this paper, mean-variance optimisation of different categories of property (or property “sectors”) is contrasted with Betas as an alternative definition of risk for a fund. The Beta compares the volatility of each sector with an index of the performance of the whole market. The Beta is the usual measure of risk in single index models and its relevance in gauging the benefits of diversification and portfolio risk is explored.

Part 3 describes the mathematical relationship, first, between sector covariances and the Betas of those sectors and, secondly, between portfolio standard deviations and portfolio Betas. From this, it is clear why portfolio variances may indicate different optimal combinations of sectors to those suggested by the sector Betas. Also, this illustrates why sector covariances and portfolio variances cannot be interpreted in the same way as portfolio Betas.

Part 4 calculates these statistics for the main non-residential property sectors using data from the Property Council of Australia Investment Performance Index. These are used to test how portfolios constructed for optimal means and variances might differ from those based upon sector Betas.

The fifth and final part of the paper draws conclusions about the viability, advantages and disadvantages of using sector Betas to guide investors and fund managers in the construction of property portfolios. The paper does not try to suggest that either method of portfolio construction is wrong but shows that Betas can be of practical help to investors and fund managers when they are deciding upon sector diversification.
Part 2: Constructing efficient property portfolios

The usual objective of portfolio management is either to maximise the expected return from the portfolio for a given level of risk or to minimise risk at a target rate of return. Typically, past returns are used as guides to future performance. The portfolio return is the value-weighted average of the rates of return of each investment in the portfolio. Risk, as seen in the volatility of the periodic returns from the portfolio, can be measured by the standard deviation of returns. The portfolio standard deviation is the weighted average of the standard deviations of each investment, reduced by their lack of correlation. Portfolios are efficient if no other combinations of properties earned a higher rate of return with no higher standard deviation.

In practice, this approach is difficult to apply to the effect on a portfolio of acquiring a single property as historic details of the property are often not available or may be heavily influenced by unusual events in the recent history of the property. It is more reliable when measuring averages from groups of similar properties (Brown, 1991: 172) but it is unclear whether these retrospective diversification gains are statistically significant (Rubens et al., 1998: 78). Mean-variance optimisation may be helpful in selecting the preferred property sector for further acquisitions or disposals although it is unlikely to be conclusive ever.

Although not all funds have a clear “top-down” strategy, most operate under some approximate guidelines for the value-weighting of different property sectors within their portfolios (Rowland, 1997: 283). They are likely to check the past returns, volatility and covariances of their own properties in different categories to judge what would have been efficient property portfolios. They may refer to the returns, volatility and covariances of a larger sample of properties, such as those in a property index and its indices of sectors and sub-sectors.

The traditional methods of diversifying real estate portfolios are by property use (such as office, retail, industrial and leisure properties) and by location. There have been many studies of the covariances (standardised to the correlation coefficients) of periodic returns between properties in different uses and in different locations. The general conclusions have been that there have been strong benefits from diversifying property portfolios by the use of the buildings. Most studies have shown the benefits of diversifying by region within one country have been less but still significant, even if the regions are defined in terms of their economic base (Eichholtz et al., 1995: 45; Hartzell et al., 1993: 8; Mueller, 1993: 65).

Many of these studies have then adopted the mean-variance framework to gauge weightings for each property sector that would have resulted in efficient portfolios at different levels of risk. The conclusions as to which sectors should be held by investors are expressed as an efficient frontier of target returns and levels of risk. The results of

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1 It is doubtful whether past property returns are truly comparable with the measures for other investment classes, largely because property returns are based upon infrequent valuations (the issues are summarised in Rowland, 1997: 282). As this paper concentrates solely on comparisons amongst different property sectors, this is largely irrelevant.

2 In a US survey of pension fund managers, 37 per cent used correlation coefficients between asset classes when determining real estate asset allocation and 24 per cent used modern portfolio theory (Worzala and Bajtelsmit, 1997: 51).

3 Lee and Byrne (1998: 38) measure risk as the mean absolute deviation and obtain similar results to conventional mean-variance optimisation.
the studies have favoured different sectors, dependent on the country, the time period and how the sectors are defined. A dramatic difference between the favoured weightings in the 1980s and in the 1990s can be observed, as well as a tendency to construct portfolios with heavy or exclusive emphasis on one or two categories of property that have performed above average.

The alternative to selecting efficient portfolios by their means and variances is to consider the means and portfolio Betas. The Beta is a measure of risk derived from the capital asset pricing model, which ignores those uncertainties which will be diversified away by holding a large portfolio of investments. The Beta is the remaining or systematic risk, expressed as a ratio of the average market volatility.

When considering sector weightings in a property portfolio, it is the systematic risk which is of concern to those investors whose portfolios include enough properties from each sector to cancel out most, if not all, of the “sector-specific” risk. There would be dangers in relying upon Betas to weight sub-sectors of a property fund which invested in only one sector. For example, a property trust specialising in shopping centres is exposed to risks that the retail sector might underperform the composite property index from which the Beta had been calculated.

However, it is also true that investors cannot rely on sector correlations calculated from large samples of properties if their own properties in each sector are not sufficiently diverse to approximate the return from the sample which has been compiled into the sector index. Recent research has noted that a property portfolio must be large and varied to be reasonably diverse and to track an index (Brown, 1997: 133; Brown and Matysiak, 1995: 34; Schuck and Brown, 1997: 173).

**Part 3: The mathematical basis of these two portfolio models**

Because these two models of portfolio risk are deceptively similar, it is helpful to contrast their mathematical formulations. This clarifies some of the potential uses and misuses of sector Betas. First, the interpretation of and relationships between sector variances, covariances, correlation coefficients and Betas are described. Then, portfolio Betas are compared with portfolio variances as measures to optimise sector weightings, in particular when adding a property or properties of one sector to a portfolio.

The covariance of the periodic returns from two sectors measures how closely one varies about its mean in comparison with the other. The covariance is the first building block of portfolio theory and can be found as:

\[ COV_{ij} = \rho_{ij} \sigma_i \sigma_j \]

where

- \( COV_{ij} \) is the covariance between the periodic returns of sector \( i \) and sector \( j \);
- \( \rho_{ij} \) is the correlation coefficient between the periodic returns of sector \( i \) and sector \( j \);
- \( \sigma_i \) is the standard deviation of the periodic returns of sector \( i \);
- \( \sigma_j \) is the standard deviation of the periodic returns of sector \( j \).

Although the covariance incorporates both a measure of correlation and the standard deviation of each sector, it is not a standardised measure, making it impossible to compare covariances between different sectors. The correlation coefficient is
standardised (between 1 and -1) but does not include a guide to the volatility of either of the sectors. Using the Markovitz model, the correlation coefficients cannot be used to judge which type of properties should be aggregated in a portfolio without knowing how volatile each sector has been.

The Beta is a measure of both correlation and relative volatilities. The Beta is normally formulated as:

$$\beta_i = \frac{COV_{im}}{\sigma_m^2}$$

where $\beta_i$ is the Beta of asset or sector $i$; $COV_{im}$ is the covariance between the periodic returns of sector $i$ and the market average or index ($m$); and $\sigma_m^2$ is the variance of the periodic returns of the market average or index.

As $COV_{im} = \rho_{im} \sigma_i \sigma_m$, the Beta can be expressed as:

$$\beta_i = \rho_{im} \frac{\sigma_i}{\sigma_m}$$

where $\rho_{im}$ is the correlation coefficient between the periodic returns of sector $i$ and of the market average or index ($m$).

And it is now evident that, first, the Beta only measures that portion of the relative volatility of the sector that can be explained by changes in the periodic returns in the market ($\rho_{im}$); and, secondly, Beta is a measure of the relative volatility of the sector and the market ($\sigma_i / \sigma_m$). As such, Betas are a convenient guide to the effects on portfolio risk of each sector, provided that the portfolio is reasonably diversified across all aspects of the market.\(^4\)

If the capital asset pricing model provides a reasonable (linear) fit of sector returns to Betas, the relationship between sector covariances to sector Betas is as follows:

$$COV_{ij} = \beta_i \beta_j \sigma_m^2$$

where $\beta_j$ is the Beta of asset or sector $j$.

This can be derived from the expectations operator (Benninga, 1997: 90; Brown, 1991: 47). If there are many sectors, this may provide a quick way of estimating the variance and covariance matrix (provided that the Betas are reasonable predictors of return, but it is probably unnecessary when considering covariances between a small number of sectors). The formulation is shown here to indicate the relationship between the two

\(^4\) It follows that, if the investor or fund is specialised, the Beta should be calculated using a subset of specialised properties to construct the index.
indicators of portfolio diversification. From this equation, the correlation coefficient between sectors \( i \) and \( j \) can be found as:

\[
\rho_{ij} = \rho_{im}\rho_{jm}
\]

In Part 4 below, the accuracy of calculating sector correlation coefficients in this way is tested on Australian data.

Turning to portfolio construction, both the portfolio standard deviation and the portfolio Beta indicate risk (to be evaluated together with the portfolio return in the search for an efficient portfolio). The following formulae can be found in many financial and portfolio management texts but are printed below to show the contrasts.

The portfolio standard deviation requires the calculation of a matrix of all the possible covariances between each sector as well as the variances of each sector, as follows.

\[
\sigma_p = \sqrt{\sum_{i=1}^{s} \sum_{k=1}^{s} w_i w_k C O V_{ij}}
\]

where \( \sigma_p \) is standard deviation of the portfolio;

\( W_i \) and \( W_k \) are the percentages by value of each of the sectors 1 to \( s \).

Although this is not complicated when the number of sectors is small, it is difficult for an investor to tell at a glance how changing sector weightings will influence risk.

The Beta of a portfolio is the value-weighted average of the sector Betas and hence it is reasonably easy to anticipate the effects of changing weightings.

\[
\beta_p = \sum_{i=1}^{s} w_i \beta_i
\]

where \( \beta_p \) is the Beta of the portfolio;

\( W_i \) is the percentage by value of each of the sectors 1 to \( s \); and

\( \beta_i \) is the Beta of sector \( i \).

To illustrate, the following formulae show the effect of adding a property or properties of sector \( i \) to an existing portfolio \( j \). Using the Markovitz model, the standard deviation of the new portfolio \( p \) is found as on the following page and it is far from obvious what the effect of the change to the portfolio risk will be.

\[
COV_{ij} = \beta_i \beta_j \sigma_m^2
\]

is the same as

\[
\rho_{ij} = \frac{\beta_i \beta_j \sigma_m^2}{\sigma_i \sigma_j}
\]

Substituting

\[
\beta_i = \rho_{im} \frac{\sigma_i}{\sigma_m}
\]

and

\[
\beta_j = \rho_{jm} \frac{\sigma_j}{\sigma_m}
\]

and simplifying this gives

\[
\rho_{ij} = \rho_{im} \rho_{jm}
\]

Page 6
\[
\sigma_p = \sqrt{w_i^2 \sigma_i^2 + (1 - w_i)^2 \sigma_j^2 + 2 \rho_{ij} w_i (1 - w_i) \sigma_i \sigma_j}
\]

where \( \sigma_p \) is the standard deviation of the new portfolio \( p \);
\( \sigma_i \) is the standard deviation of sector \( i \) from which the additional properties come;
\( \sigma_j \) is the standard deviation of the existing portfolio \( j \); and
\( w_i \) is the percentage by value of the new portfolio that comprises the additional properties in sector \( i \).

The Beta of the new portfolio is found as:
\[
\beta_p = w_i \beta_i + (1 - w_i) \beta_j
\]

where \( \beta_p \) is the Beta of the new portfolio \( p \); and
\( \beta_j \) is the Beta of the existing portfolio \( j \).

The change in the portfolio Beta is \( w_i (\beta_i - \beta_j) \) and the effects of adding properties from one (or more) sector to the portfolio are evident.

Advantages of using sector Betas to gauge portfolio risk (rather than the portfolio standard deviation) are the simplicity of calculation and the ease of anticipating the effects of changing sector weightings. Betas may be valid when one or more of the assumptions of mean-variance analysis are breached (Lee and Byrne, 1998: 39). However, it must not be overlooked that portfolio Betas would understate risk for funds which do not have a reasonable spread of investments across all sectors of the index. Using Betas for comparison of sectors or sub-sectors, the index should have a similar breadth as the fund.\(^6\)

The Betas of sectors or sub-sectors of the property market compare the relative risks of each sector. However, if much of the risk is specific to one sector, that sector will have a low (or even negative) correlation with the market and little of the volatility of the sector will show up in its Beta. In contrast, the high specific risk will increase the portfolio standard deviation, even though the sector may have low or negative correlations with most other sectors. This is the main reason why portfolio standard deviations and Betas may suggest different sector weightings.

**Part 4: Empirical tests of Betas for weighting property sectors**

In this part of the paper, the periodic returns from the Property Council of Australia Investment Performance Index are used to test whether sector and portfolio Betas give meaningful guidelines for investors. It is assumed that the investor has a diversified portfolio of properties and is seeking guidance as to which sector or sectors to expand without taking unnecessary risk. This is the most likely occasion when portfolio models might be used by investors or fund managers.

\(^6\) Single index models using a narrow index of one sector provide insights into sub-sector weightings but the results are not relevant to the concept of a broad “market portfolio” and “market risk”, as envisaged by the capital asset pricing model (Van Horne, 1997: 74).
Table 1 presents the main characteristics of the Property Council of Australia Investment Performance Index, as at June 1998, including the percentage by value of the sectors and sub-sectors used in the tests (all the tables and graphs can be found at the end of the paper). Table 2 displays annualised rates of return and a variety of measures of risk, all based upon returns over each 6 month period between December 1984 and June 1998. It is evident that retail properties have considerably outperformed the other three sectors. The retail sector showed higher annual returns, lower volatility (as measured by both the standard deviation and the coefficient of variation) and also the lowest correlation coefficients with other sectors.

The Beta has been calculated using the composite property index as the market average. It follows that, because Beta is a combined measure of correlation and volatility, the retail sector has the lowest sector Beta. With such clear dominance by one sector, it is impractical to test whether Betas provide a suitable guide to portfolio construction. The retail sector during this period dominated all the others and efficient portfolios would contain retail properties only. The ranking of each sector by risk is the same by their standard deviations and their Betas, being the reverse of the expected ranking by rates of return. This inconsistency with modern portfolio theory (which assumes higher returns will go to those who take more risk) undermines the confidence of investors who might otherwise rely on past performance to guide future portfolio structure.

A further criticism of practical applications of portfolio theory is that the statistics are often unstable over different eras. Whilst it might be anticipated that rates of return would change with the state of the market, portfolio diversification relies upon lasting patterns in volatility and covariance. If standard deviations and correlation coefficients of sectors are unstable, mean-variance optimisation will suggest major portfolio revisions regularly. This is not viable for property investments because of the transaction costs. Any measures of performance which are less variable during different states of the market instill more confidence in investors.

Tests of the relative stability of sector means, standard deviations, correlation coefficients and Betas were carried out by adopting a rolling 10 year period for analysis. Table 3 confirms that the Betas are less variable over these periods than the standard deviations, correlation coefficients and covariances. The average coefficient of variation of the changing Betas over time was 0.038, compared with 0.182 for the covariances. The smaller variability may be because Betas are calculated from a composite index of performance rather than from the relationships between individual sectors. Graphs 1 to 4 that returns, standard deviations and correlation coefficients have generally declined in more recent ten year periods but curiously Betas have increased slightly.

A further test of the stability of these portfolio models over time is to compare the effects of changing the portfolio weightings on the portfolio return, the portfolio standard deviation and the portfolio Beta. Table 4 displays the results of substituting 10 per cent of a composite property portfolio with additional properties in each sector. In each of the 10 year rolling periods, the change in return, standard deviation and Beta might be different. The variation in those differences, as expressed as their standard deviations is shown for each sector in Table 4. The absolute variation is shown and, on the next row, this is expressed as a percentage of the sector average variation. The percentage variation for the Betas (0.29%) is less than those of the standard deviations (0.48%), again suggesting that the Beta provides a more stable portfolio measure.  

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7 It is unlikely that any of these differences would be statistically significant.
In Part 3 above, it was shown how, if the return from each sector has a linear relationship with its Beta, the correlation coefficient can be calculated from the sector Betas or more simply,

\[ \rho_{ij} = \rho_{im} \rho_{jm} \]

The sector correlation coefficients calculated from this equation are displayed in Table 5 (with the coefficients calculated directly from the covariance of each sector in brackets). Considerable differences are evident and it would seem that, for this data, Betas are an unreliable way of deriving correlation coefficients for each sector. This is not surprising given the overwhelming conclusion from tests of the capital asset pricing model that either it is a poor predictor of returns or the market index is defined incorrectly (Fama and French, 1992: 427; Roll, 1977: 129).

Because of the difficulties of using portfolio models with data that does not conform to expected risk and return relationships, further analysis has been carried out on the sub-sectors of retail properties in the Property Council of Australia Investment Performance Index. Sub-sector periodic returns are published for four states only but the composite retail index does include a small number of properties from elsewhere.

The same statistics have been calculated for the sub-sectors as for the sectors in Table 2. Past returns from the retail sub-sectors in Australia show a pattern of volatility and average returns that is more typical. These are displayed in Table 6, together with the matrix of correlations and covariances and the Betas of the sub-sectors. Betas were calculated using the composite property index as the market average. Because the retail sector has been considerably less volatile than the composite property index, all the Betas are well below 1.

The standard deviations and Betas rank the sub-sectors differently. However, when the sub-sector correlation coefficients are considered, the standard deviations are largely consistent with the sub-sector Betas. One of the sub-sectors (Victoria) with a low Beta has a high standard deviation, which can be explained by the low correlation coefficients between Victoria and the other sub-sectors. The strong diversification benefits of the Victorian retail sub-sector compensate for its low returns. Using either mean-variance or mean-Beta measures of performance, an investor looking for a low-risk retail region would select Victoria based upon its past returns.

Queensland retail properties would also be attractive based upon either their low standard deviation or low Beta, despite their poorer diversification benefits. WA retail properties appear suited only to less risk-averse investors. Based upon its risk-adjusted performance (using either its standard deviation or Beta), the NSW retail sector has been the weakest sub-sector of the four.

The next comparisons of the statistics in this sub-sector involved constructing optimal portfolios at different levels of risk using mean-variance and using mean-Beta criteria. Mean-variance optimisation (without short sales) was achieved using an Excel Solver routine for the variance/covariance matrix, as described by Benninga (1997: 127). This routine maximises Theta, being:

\[ \frac{R_p - C}{\sigma_p} \]

where \( R_p \) is the portfolio average return;
\( \sigma_p \) is the portfolio standard deviation; and

\( C \) is a constant, varied in successive calculations to achieve portfolios of different levels of risk.

Mean-Beta optimisation was achieved with a similar Excel Solver routine to minimise Betas for given portfolio average returns. The shapes of the two efficient frontiers can be compared in Graph 5. The efficient frontier found by mean-variance optimisation is a gradual curve over a short range of returns. The frontier found by mean-Beta optimisation has two straight sections, reflecting the additive properties of sector Betas. Nevertheless, the frontiers are not greatly different.

The portfolio weightings suggested as optimum at four different points on the two frontiers are compared in Graph 6 (MV for the mean-variance frontier and B for the Beta-frontier). At the higher target returns (15.4 and 15.9 per cent per annum), there is virtually no difference between the weightings, with heavy reliance on Western Australian retail properties for risk-tolerant investors and more Queensland retail properties for those who are more risk averse. At lower target returns (14.5 and 14.9 per cent per annum), the frontier based on Betas introduces more Victorian properties than the mean-variance frontier, which favours New South Wales properties. A Beta-frontier will never use sectors that are “inferior”, in the sense that they show a higher risk for less return. This is because the portfolio Beta is a weighted average of the sector Betas. A mean-variance frontier may introduce sectors that appear inferior in isolation but are redeemed by their lack of correlation with the other sectors.

Mean-variance optimisation is often criticised because it suggests portfolios which rely heavily or exclusively on one or two categories of property that have performed above average. This is because the portfolio returns and standard deviations are determined largely by the weightings in each sector, with the lack of correlation having limited impact on the portfolio risk. These tests suggest that mean-Beta optimisation has exactly the same tendency. To overcome this shortcoming, users of portfolio models impose constraints upon the sector weightings.

Mean-variance and Beta-frontiers were compared using constraints on the retail portfolio (NSW restricted to between 30 and 60 per cent of the value of the portfolio; Victoria and Queensland between 20 and 40 per cent; and Western Australia between 10 and 30 per cent). The efficient frontiers with these constraints are displayed in Graph 7. The constraints lower and limit the range of returns on both frontiers but otherwise their characteristics are similar to those with unconstrained weightings.

The portfolio weightings for the two frontiers are compared at four different levels of return in Graph 8. At the highest target return (14.65 per cent per annum), there is virtually no difference between the weightings, with heavy reliance on Western Australian retail properties and the minimum for each other State. At the 14.5 per cent per annum target return, Queensland properties replace some of the Western Australian ones. are substituted for At lower target returns (14.0 and 14.25 per cent per annum), the frontier based on Betas introduces more Victorian properties, whereas the mean-variance frontier increases the weighting of Queensland properties to the maximum (with the minimum for each other State).

The Beta-frontier (with and without constraints) favours Victorian retail properties at lower levels of risk. The Victorian retail sector has a much lower correlation with the property index than the other States (see Table 6). As noted above, this offsets the
effects of its high standard deviation, which is more important to the mean-variance frontier.

These tests with Australian data suggest that the results from using the mean-variance framework to establish property sector weightings are broadly similar to the results based on the mean and Beta of portfolios. However, the differences are large enough to warrant further investigation with different sets of data and some more practical trials of how investors and fund managers would cope with different measures to help them construct diversified portfolios. The data gives no support to the adoption of the capital asset pricing model to determine required rates of return.

**Part 5: Conclusions**

Diversification within a real estate portfolio is a major consideration for most fund managers when they are deciding what type of property or region should be the focus of the search for further property acquisitions. In recent years, some fund managers have used mean-variance optimisation models to help with these decisions.

There appear to be no theoretical reasons why they should not use a single index model in place of sector correlations and portfolio standard deviations. The Beta is the best known measure of volatility and correlation with a single index (although there may be other formulations that are appropriate). Both variances and Betas measure related aspects of risk but their derivations may lead to differing conclusions about appropriate sector weightings.

Tests of Australian data confirm that broadly similar conclusions are drawn from mean-variance and Beta-frontiers. Betas for sectors and sub-sectors are valid measures only if the investor holds a sufficiently diversified portfolio to ignore sector-specific risk. However, investors can only rely on sector correlations calculated from large samples of properties if their own properties in each sector are sufficiently diverse to approximate the return from the sample compiled into the sector index. Recent research has noted that a property portfolio must be large and varied to be reasonably diverse and to track an index (Brown, 1997: 133; Brown and Matysiak, 1995: 34; Schuck and Brown, 1997: 173).

Betas are easier to interpret and calculate and these tests suggest that they may be slightly less variable over time than portfolio variances. Portfolio standard deviations and portfolio Betas are superficially similar measures of risk but they do not serve the same functions. The use of a single index model to evaluate property sectors and weightings is worthy of further investigation.
References
### Tables and graphs

#### Table 1: Sample Characteristics

<table>
<thead>
<tr>
<th></th>
<th>Total value of the 642 sample buildings</th>
<th>Data analysed in 6 monthly periods</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AUS$38,492 million at June 1998</td>
<td>From June 1984 to June 1998</td>
</tr>
</tbody>
</table>

**Sample by sectors**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Percentage by value</th>
</tr>
</thead>
<tbody>
<tr>
<td>203 CBD office buildings</td>
<td>45.3%</td>
</tr>
<tr>
<td>116 Non-CBD offices</td>
<td>9.5%</td>
</tr>
<tr>
<td>133 retail properties</td>
<td>38.2%</td>
</tr>
<tr>
<td>190 industrial properties</td>
<td>7.0%</td>
</tr>
</tbody>
</table>

**Sub-sectors adopted**

<table>
<thead>
<tr>
<th>Sub-sector</th>
<th>Percentage by value</th>
</tr>
</thead>
<tbody>
<tr>
<td>N.S.W. retail (49 buildings)</td>
<td>13.9%</td>
</tr>
<tr>
<td>Victorian retail (30 buildings)</td>
<td>8.7%</td>
</tr>
<tr>
<td>Queensland retail (28 buildings)</td>
<td>8.7%</td>
</tr>
<tr>
<td>W.A. retail (14 buildings)</td>
<td>3.6%</td>
</tr>
</tbody>
</table>

#### Table 2: Annualised return and risk by sector

**December 1984 to June 1998**

<table>
<thead>
<tr>
<th>Sector</th>
<th>Composite</th>
<th>CBD offices</th>
<th>Non-CBD offices</th>
<th>Retail</th>
<th>Industrial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>10.02%</td>
<td>7.94%</td>
<td>8.66%</td>
<td>14.10%</td>
<td>11.17%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>7.61%</td>
<td>9.84%</td>
<td>8.37%</td>
<td>4.16%</td>
<td>6.55%</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.76</td>
<td>1.24</td>
<td>0.97</td>
<td>0.29</td>
<td>0.59</td>
</tr>
</tbody>
</table>

**Matrix of correlation coefficients** (with variances or covariances in brackets)

<table>
<thead>
<tr>
<th></th>
<th>Composite property</th>
<th>CBD offices</th>
<th>Non-CBD offices</th>
<th>Retail</th>
<th>Industrial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Composite property</td>
<td>1 (0.58)</td>
<td>0.99 (0.74)</td>
<td>0.83 (0.53)</td>
<td>0.77 (0.24)</td>
<td>0.85 (0.42)</td>
</tr>
<tr>
<td>CBD offices</td>
<td>1 (0.97)</td>
<td>0.81 (0.67)</td>
<td>0.73 (0.30)</td>
<td>0.84 (0.54)</td>
<td></td>
</tr>
<tr>
<td>Non-CBD offices</td>
<td>1 (0.70)</td>
<td>0.42 (0.15)</td>
<td>0.90 (0.49)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Retail</td>
<td>1 (0.17)</td>
<td>0.46 (0.13)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
<td>1 (0.43)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sector Betas**

| Sector Betas | 1     | 1.29  | 0.91  | 0.42  | 0.73  |
Table 3: Variability of sector statistics
10 year periods between December 1984 and June 1998

<table>
<thead>
<tr>
<th></th>
<th>CBD offices</th>
<th>Non-CBD offices</th>
<th>Retail</th>
<th>Industrial</th>
<th>Overall variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest/highest returns</td>
<td>3.09/8.05%</td>
<td>6.00/7.47%</td>
<td>12.39/15.77%</td>
<td>8.99/9.84%</td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.29</td>
<td>0.07</td>
<td>0.08</td>
<td>0.03</td>
<td>0.118</td>
</tr>
<tr>
<td>Lowest / highest st.dev.</td>
<td>8.77/11.4%</td>
<td>8.27/9.54%</td>
<td>3.85/4.59%</td>
<td>6.85/7.39%</td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.09</td>
<td>0.05</td>
<td>0.06</td>
<td>0.02</td>
<td>0.055</td>
</tr>
<tr>
<td>Lowest / highest Betas</td>
<td>1.27/1.29</td>
<td>0.93/1.00</td>
<td>0.38/0.44</td>
<td>0.74/0.87</td>
<td></td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.06</td>
<td>0.038</td>
</tr>
</tbody>
</table>

Table 4: Variability of portfolio statistics
10 year periods between December 1984 and June 1998

<table>
<thead>
<tr>
<th>Standard deviation of the changes</th>
<th>CBD offices</th>
<th>Non-CBD offices</th>
<th>Retail</th>
<th>Industrial</th>
<th>Overall variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return - absolute</td>
<td>0.03%</td>
<td>0.12%</td>
<td>0.02%</td>
<td>0.11%</td>
<td></td>
</tr>
<tr>
<td>% of sector average</td>
<td>0.57%</td>
<td>1.68%</td>
<td>0.16%</td>
<td>1.18%</td>
<td>0.80%</td>
</tr>
<tr>
<td>St. dev. - absolute</td>
<td>0.02%</td>
<td>0.05%</td>
<td>0.03%</td>
<td>0.06%</td>
<td></td>
</tr>
<tr>
<td>% of sector average</td>
<td>0.15%</td>
<td>0.51%</td>
<td>0.78%</td>
<td>0.80%</td>
<td>0.48%</td>
</tr>
<tr>
<td>Beta -absolute</td>
<td>0.0008</td>
<td>0.0031</td>
<td>0.0021</td>
<td>0.0050</td>
<td></td>
</tr>
<tr>
<td>% of sector average</td>
<td>0.06%</td>
<td>0.32%</td>
<td>0.50%</td>
<td>0.65%</td>
<td>0.29%</td>
</tr>
</tbody>
</table>
Table 5: Comparison of correlation coefficients

**Figures in bold type are from linear estimator of Beta**

*(figures in brackets are coefficients calculated directly from the sectors)*

<table>
<thead>
<tr>
<th>Sector</th>
<th>CBD offices</th>
<th>Non-CBD offices</th>
<th>Retail</th>
<th>Industrial</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBD offices</td>
<td>1</td>
<td><strong>0.82</strong> (0.81)</td>
<td><strong>0.77</strong> (0.73)</td>
<td><strong>0.85</strong> (0.84)</td>
</tr>
<tr>
<td>Non-CBD offices</td>
<td>1</td>
<td><strong>0.64</strong> (0.42)</td>
<td><strong>0.70</strong> (0.90)</td>
<td></td>
</tr>
<tr>
<td>Retail</td>
<td></td>
<td>1</td>
<td><strong>0.66</strong> (0.46)</td>
<td></td>
</tr>
<tr>
<td>Industrial</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Table 6: Annualised return and risk by sub-sector

*December 1984 to June 1998*

<table>
<thead>
<tr>
<th>Sub-sector</th>
<th>Composite retail</th>
<th>NSW retail</th>
<th>Victorian retail</th>
<th>Queensland retail</th>
<th>W.A. retail</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return</td>
<td>14.10%</td>
<td>14.33%</td>
<td>12.79%</td>
<td>14.79%</td>
<td>16.12%</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>4.16%</td>
<td>5.13%</td>
<td>6.15%</td>
<td>4.34%</td>
<td>5.65%</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.29</td>
<td>0.36</td>
<td>0.48</td>
<td>0.29</td>
<td>0.35</td>
</tr>
</tbody>
</table>

**Matrix of sub-sector correlation coefficients** *(with variances and covariances in brackets)*

<table>
<thead>
<tr>
<th>Composite property</th>
<th>NSA retail</th>
<th>Victorian retail</th>
<th>Queensland retail</th>
<th>W.A. retail</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.77</strong> (0.24)</td>
<td>1 (0.26)</td>
<td>0.50 (0.16)</td>
<td><strong>0.75</strong> (0.17)</td>
<td>0.59 (0.17)</td>
</tr>
<tr>
<td>NSW retail</td>
<td><strong>0.78</strong> (0.31)</td>
<td>1 (0.38)</td>
<td><strong>0.61</strong> (0.16)</td>
<td><strong>0.44</strong> (0.15)</td>
</tr>
<tr>
<td>Victorian retail</td>
<td>1 (0.38)</td>
<td><strong>0.61</strong> (0.16)</td>
<td>1 (0.19)</td>
<td><strong>0.67</strong> (0.16)</td>
</tr>
<tr>
<td>Queensland retail</td>
<td>1 (0.19)</td>
<td><strong>0.61</strong> (0.16)</td>
<td>1 (0.32)</td>
<td></td>
</tr>
<tr>
<td>W.A. retail</td>
<td>1 (0.32)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Sub-sector Betas**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>0.42</strong></td>
<td><strong>0.53</strong></td>
<td><strong>0.37</strong></td>
<td><strong>0.38</strong></td>
<td><strong>0.50</strong></td>
</tr>
</tbody>
</table>
Graph 1

Changing average returns

Graph 2

Changing volatility
Graph 3

Changing correlation coefficients

- CBD offices & retail
- CBD & Non-CBD offices
- CBD offices & industrial
- Retail & Non-CBD offices
- Retail & industrial
- Non-CBD offices & industrial

Graph 4

Changing Betas

- Composite property
- CBD offices
- CBD offices & retail
- Non-CBD offices
- Retail
- Industrial

10 year period ending
Graph 5

Efficient frontiers (unconstrained weightings)

Return vs. Standard deviation

Graph 6

Portfolio weightings (unconstrained)

Target return

# = not efficient
Graph 7

Efficient frontiers (constrained weightings)

Graph 8

Portfolio weightings (constrained)