EQUILIBRIUM OPTION PRICING MODEL UNDER A SPECIFIC SYSTEM: A VIEW ON CHINA'S HOUSING BOOM

GUO XIAOYANG AND LIU HONGYU
Tsinghua University

ABSTRACT
For real estate markets, especially emerging economies with a specific system, it is essential to focus on the reasonableness of the theoretical hypothesis underpinning the real option pricing model, prior to considering application in valuation and investment.

In this paper, we focus on this first-step issue in the context of the Land Finance System in China. Compared with the classic hypothesis, for which the underlying housing price is exogenous to the value of the to-be-developed vacant land, the Land Finance System in China impairs this exogeneity to some extent by the interaction between land value and housing market price.

Considering such relationships, we expand the classic option pricing model into a succinct equilibrium model where both the land value and the housing market price are determined simultaneously. The effect of the Land Finance System is reflected as a parameter β, similar to the parameter ω that reflects the housing market volatility.

Numerical analysis is conducted to investigate the nature of the effect of the system parameter β and the market parameter ω on the variations of both the expected land value and housing market price. Finally, conclusions are drawn concerning an explanation of the housing boom in terms of a system effect.

Keywords: real option, land valuation, housing market, Land Finance System, China

INTRODUCTION
There has been extensive research concerning land pricing and development strategy problems in the context of real options. Since about thirty years ago, most of the relevant research has focused on issues of land valuation, the optimal development time and scale and urban spatial patterns (Titman 1985; Capozza et al 1990; Williams 1991, 1993; Quigg 1993; Capozza et al 1994; Rhys et al 2002; Rocha et al 2007; Grovenstein et al 2011). Empirical research, although much later, however, has preferred to test the existence of the real option, the decision-making characteristics of real options and the determinants of the option properties (Holland et al 2000; Harchaoui et al 2001; Sing et al 2001; Yamazaki 2001; Cunningham 2006; Bulan et al 2009).

Three critical properties have been identified concerning land and the intrinsic value of options (Titman 1985; Mcdonald et al 1986; Abel et al 1996; Krychowski et al 2010).The first is the *expendability* of land investment. Most urban land is purchased for further construction no matter what the building type is to comply with the planning constraint or how long the waiting time is; the flexibility of land development provides the land with the essence of a real option. The second is the *irreversibility* of land investment. Because of the physical nature and durability of buildings, they cannot be removed once the construction process starts. This high recovery cost makes the land investment almost irreversible in the short run. The last point is the *uncertainty* of land investment. The uncertainty comes from the market price of the underlying property. Generally, the market price of underlying property is seen as exogenous and the value of the option is determined by the price and the volatility of the underlying property.

However, this relationship between land value and the price of underlying property is impaired under the Land Finance System in China, especially for land used for urban residential. The so-called Land Finance System is, for short, a system of local government finance for urban construction and development through land transfer income. The system comprises two aspects. One is that the land transfer income has become large scale and an important complement for the regular revenue for local governments. The other aspect is that the urban construction fund has also been strongly relying on the land transfer income.

Why cannot Chinese local governments finance the urban construction fund by issuing bonds, just like those in U.S. cities? By the regulation of the law, only the central government has the authority to issue bonds in China, while local governments are limited to raise money through financial instruments for urban construction, which are regarded as a common way in most developed countries. Meanwhile, it is strongly believed that the current two-tier finance system in China does not adequately balance the property rights and the powers between central and local government, which usually results in local governments finding it hard to gain enough money for urban development through normal revenue including taxation and related administrative fees. Under the goal and the pressure of development, local governments rely on land transfer income.

Gyourko et al (1999), Zheng et al (2008) and Ding et al (2010), among others, have noted the improvement of urban amenities and opportunities, referring to the number and quality of the commuting facilities, the hospitals, the schools, the leisure facilities and the jobs and learning opportunities which will positively affect the housing price, essentially via the fostered utilities and therefore theoretically improved demand. Because the urban construction fund for such amenities and facilities is largely supported by the land transfer income in China, the housing price, therefore, will be affected "conversely" to some extent by the land value compared to the classic hypothesis of real options.

Obviously, this effect will not stop since a change in housing price will consequently render a change in the land value again, which is an interactive loop behind but what we can see seemingly is only the equilibrium result. Therefore, when using an option pricing model for land valuation in China's urban housing market, we will undoubtedly encounter the endogenous problem between housing price and land value and have to adjust the theoretical pricing model. Here, we only focus on those lands that are due to be used as residential. We amend the housing price determination condition of the real option model as a function of the land value following some specific forms and constraints. After such adjustment, an equilibrium option pricing model with explicit mathematical expression can be established.

It is particularly interesting that, under this equilibrium option model, both the factors of the Land Finance System and the housing market volatility affect the housing price itself, which can be never revealed via the classic real option model due to the exogenous housing price hypothesis. The positive effect of those factors was predicted on the housing price under some conditions through numerical analysis, providing incidental results with which to view the continually rising housing price in China. With such activity happening in most of China's cities, including the rapid development of urban facilities and amenities, the large amount of old-zone demolitions, the subsequent transfer of the rearranged land and the soaring housing price, the new model proposed may show that the housing boom in China may have a strong relationship to the system, inferring a systematic determinant in housing price under the Land Finance System. Furthermore, under that interactive loop between land value and housing price triggered by the system, the housing market volatility can also affect the housing price itself, which could be significant for an emerging market like China.

This paper is structured as follows. Following the modelling discussion, a series of numerical analyses are undertaken to investigate the relationship between housing price and land value under the Land Finance System and, furthermore, to demonstrate the effects of two critical parameters, the Land Finance System factor and the market volatility factor, on the valuation. Following consideration of results and findings, conclusions are then drawn.

THEORETICAL MODEL

Hypothesis Adjustment for China

Classic Hypothesis of Exogenous Underlying Asset

In the classic real option theory, the price of the underlying asset is assumed to follow the stochastic process of geometric Brownian motion in the form as Equation 1. In this mathematic formula, μ is the drift parameter indicating the marginal change of the asset price in terms of instantaneity or the expected return for long run. The parameter σ is the variance parameter indicating the volatility of the asset price and B(t) is the standard Brownian motion. According to the different types of assets, the parameter μ could appear as different expression, for example, the constant typically. In the realm of real estate, alike the financial asset, μ is always set in the form of *(g-d)*, where *g* is the capital gains and *d* is the dividend for developed property such as the rental income.

$$dP / P = \mu_p dt + \sigma_p dB \qquad \text{Equation 1}$$

No matter how complex the mathematic expression is, the core concept in the classic real option theory is that the price of underlying asset is exogenous—the price of the underlying asset runs as a path that follows the geometric Brownian motion, and the option value is unilaterally determined by the price of the underlying asset.

Cost is another critical determinant of the option value with a similar stochastic expression as Equation 2.

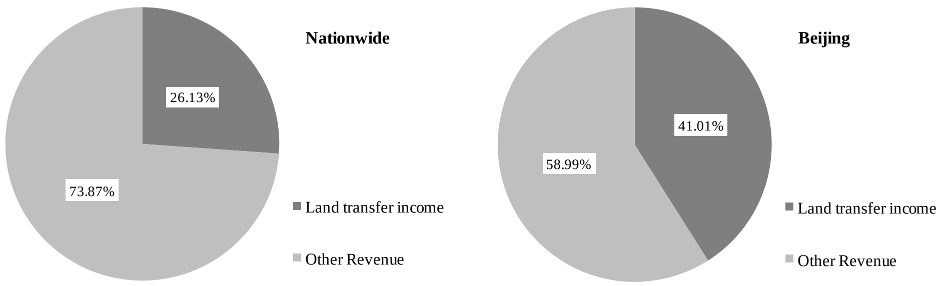
$$dC / C = \mu_c dt + \sigma_c dB \qquad \text{Equation 2}$$

Accordingly, in the housing market, the classic hypothesis assumes the housing price and development cost as exogenous following the geometric Brownian motion, while the land price is endogenously determined by the housing price and the cost, as an option in deciding when and how to develop.

Background of the Land Finance System in China

However, the reasonableness of the exogeneity assumption is challenged in China’s urban housing market due to the Land Finance System. In China, city or county government is the monopoly seller in the land market and has the authority to determine how to use the land transfer income. In short, the Land Finance System indicates that urban development funds strongly rely on the land transfer income through the activities of the local government.

On one hand, the land transfer income takes a relatively big place in the whole revenue of the local government, which contributes to the significance of the effect that the Land Finance System has. Firstly, the current central-local two-tier public finance system does not adequately balance the property rights and the powers between central and local governments, which usually results in the shortage of urban development funds for most latecomer locations, where industry and commerce are not well developed. Secondly, with the economic growth and the uninterrupted urbanization process, the value of urban and suburban land keeps rising, which is gradually recognized by the local governments, who are the monopoly manager of local land, as an important channel to extend the government income. Figure 1 shows the proportion of land transfer income in whole of government revenue. The left chart illustrates that the nationwide land transfer income amounted to 26.13% of total local government annual revenue in 2010, at the same time the proportion even rose to 41.01%, an increase of nearly a half, in Beijing, as the right chart shows.



Proportion of Land Transfer Income in Total Revenue
Source: Authors
Figure 1

On the other hand, in terms of the expenditure, the most part of the land transfer income, after covering the cost expenditures, is used in urban construction by local government, which determines the essence of the effect that the Land Finance System takes. The nationwide situation is shown in Table 1, where the total expenditure of the land transfer income amounts to more than 90% from 2008 to 2010 (D line), of which cost expenditures take the greater proportion as about 50% (E line) and urban construction takes the second place as about 28% (F line).

No.	Items	2008	2009	2010	Total
A	Land Transfer Income	10375.28	14239.70	29109.94	53449.98
B	Expenditure of Land Transfer	10172.50	12327.10	26975.79	49475.39
B1	of which：Cost expenditure	4664.37	6695.18	13395.6	20090.78
B2	**Urban construction**	3035.32	3340.99	7531.67	13907.98
B3	Rural development	369.88	910.66	2248.27	3528.81
B4	State owned enterprise reform	1561.28	1378.27*	3336.63	4897.91
B5	Low-rent housing construction	141.65		463.62	605.27
C=B2+B3+B4+B5	Sum. of none-cost expenditures	5508.13	5631.92	13580.19	24720.24
D=B/A	Outcome-Income Ratio	98.05%	86.57%	92.67%	92.56%
E=B1/B	Cost Exp.-Total Ratio	45.85%	54.31%	49.66%	50.04%
F=B2/B	Urban Cons.-Total Ratio	29.84%	27.10%	27.92%	28.11%
G=B2/C	Urban Cons.-Non-Cost Ratio	55.11%	59.32%	55.46%	56.26%

*This number involves both State owned enterprise reform and low-rent housing construction payments.

Nationwide Income and Expenditure of Land Transfer (in RMB 0.1 Billion)
Source: Annual Report of Budget Enforcement, Ministry of Finance
Table 1

Actually, the *Act for Balance and Expenditure of State Owned Land Transfer* issued by the Ministry of Finance in 2006 authorizes the local governments to universally manage the land transfer income in the form of government funds. It indicates the use range of land transfer income, which includes the resettlement compensation, the land arrangement cost, the urban construction expenditure, the rural development expenditure, the state owned enterprise reform subsidies and the low-rent housing construction expenditure. Among these kinds of payments, the first two belong to the essential cost expenditures, which are the necessary payments for the preliminary land development. All the other payments are flexible, corresponding to the urban development strategy and government activities.

From the final line (G) of Table 1, we can see that within the flexible expenditure, urban construction always takes the larger proportion, being far more than half of the whole. The number is even higher in a big city, for instance, Beijing, as shown in Table 2. In 2009, the Beijing municipal government expended 31.95 billion RMB yuan on urban construction, which amounts to 64.65% of the total expenditure, even more than the land development related cost expenditure.

Expenditures	Amount (RMB 0.1 billion)	Proportion
Cost expenditure	107.20	21.69%
Urban construction	319.46	64.65%
Low-rent housing construction	38.50	7.79%
Rural development	26.50	5.36%
State owned enterprise reform	2.51	0.51%
Total	494.17	100.00%

Expenditure of Land Transfer Income in Beijing, 2009 (in RMB 0.1 Billion)
Source: http://news.xinhuanet.com/house/2010-03/19/content_13204816.htm
Table 2

Such data intuitively illustrates the pattern of the Land Finance System in China. This system is actually a kind of financing for local governments but without the repayment obligation of any capital and interest, contrasting with the debt-financing model used by the U.S. cities. As a result, this low-cost way of financing makes China's local governments highly reliant on the land transfer income to provide urban construction funds.

Hypothesis of Endogenous Housing Price

Concerning the expectation of the housing price distribution in any time point, a log-linear function is always used (Case et al 1991; Quigley 1995). Quigg (1993) and Grovenstein et al (2011) adopt a simple expression of the underlying asset as $P = q^{\varphi}\varepsilon$ in real option model analysis, where q is the total size in square metres, φ is the scale elasticity and ε is all the other property attributes.

In order to include the impact of the Land Finance System on the housing price function, we expand the housing price function given above by further unfolding the variable vector ε. As suggested in related studies about housing dynamics, the location attributes of housing are as important a determinant of housing price as the physical attributes (Case et al 1987; Case et al 1991; Quigley 1995) and, to the former factor, the quantity and the quality of the facilities and the amenities surrounding the housing play a very significant role in location value, theoretically, either through the accessibility to the services that are supplied by such facilities (Cheshire et al 1995; Kim et al 2005; Hess et al 2007; Zheng et al 2008) or the potential opportunities of jobs or learning (Aslund et al 2009; Fisher et al 2009; Ding et al 2010).

Therefore, as China's local governments expend a lot of the revenue that comes from the land transfer income on the city development construction, (for example, improving facilities such as the hospital or the subway line), the location value of the housing that would take advantage of completed nearby construction will be affected. As the result, the housing price function can be expressed as $P = \alpha X_1^{\gamma_1} X_2^{\gamma_2} \varepsilon$ [9], where X_1 represents the physical attributes vector, X_2 represents the location attributes vector, γ_i represent the elasticity of the attributes and ε is the error term. A change in either of the quantity or the quality will affect the value of location attributes, $X_2^{\gamma_2}$, otherwise the physical attributes.

To further format the price function involving the effect of land transfer, we should firstly more clearly define "the land", "the housing" and "the way" that land transfer income is spent on urban facilities as key assumptions. In reality, because any local government has a relatively broad administrative area, all the land transfer income generated by the land that within this boundary is used compositely for only some target facilities over the city. Without loss of the generality, we assume any land is valued according to the market-average housing price, and any transferred land based on its value has an impact on the average housing price. For the reason that there might be more than one land parcel being transferred at one time, although usually we only focus on one land parcel for the valuation and decision analysis, all the land transferred will influence the average housing price and so may result in a larger change in the average housing price and therefore a larger change in a specific land parcel than may occur if the average price was only affected by the specific land. To deal with this market-wide issue, an impact strength parameter, s, is introduced within the definition:

$$s_i = \delta_M / \delta_i \qquad \text{Equation 3}$$

where δ_M is the total impact of all the land transferred in the market on the location attributes of the market average housing price and δ_i is the impact of the specific land parcel. δ_i is given by $\delta_i = L_i^{\beta}$, where Li is the transferred income of the government from the specific land and β is the impact elasticity of the Land Finance System. This assumption is reasonable because the government has differing inclination of expenditures upon differing values of the transferred lands because the income from land transfer is finite rather than being continued. A bigger β means a larger impact of transferred land value on the average housing price so that the Land Finance System appears to have a strong impact of the interactive loop between land value and the housing market price.

Additionally, considering the value interval of β, we assume the total impact of Land Finance System on the average housing price to be a multiple effect, which leads to the expected housing price function in Equation 4:

$$\begin{aligned} E(P \mid \psi) &= \alpha \cdot X_1^{\gamma_1} \cdot (\delta_M X_2^{\gamma_2}) \\ &= \alpha \cdot X_1^{\gamma_1} \cdot (s \cdot \delta_i X_2^{\gamma_2}) \\ &= s \cdot E(L \mid \psi)^{\beta} \cdot \alpha \cdot X_1^{\gamma_1} \cdot X_2^{\gamma_2} \end{aligned} \qquad \text{Equation 4}$$

Clearly, if β equalled 0, which means that the land transfer income isn't used for urban facilities improvement and therefore there isn't Land Finance System impact, s would automatically become 1 due to Equation 3 and the whole function would return back to the common housing price.

Considering the generality, we further arrange this function, in order to simplify the final model, as Equation 5:

[9] To simplify the expression, here we define $X^{\gamma} \equiv \prod_{i=1}^{n} x_i^{\gamma_i}$

$$E(P \mid \psi) = \alpha s X^{\gamma} E(L \mid \psi)^{\beta}$$ Equation 5

Theoretical Model

Based on the previous analysis, we can then establish the equilibrium land valuation model. The housing price is assumed to follow the geometric Brownian motion but with the drift parameter, μ, also to be a stochastic variable, as in Equation 6. However, in our model the building cost is to be assumed as a constant, as in Equation 7, for two reasons. First, in reality building costs vary little both in cross section and in time series in China because of sufficient competition and stable manufacturing in the building material market; second, the constant assumption of building cost will greatly simplify the mathematical expression of the endogenous valuation model:

$$dP / P = \mu(L, X)dt + \sigma dB$$ Equation 6

$$dC / C = 0$$ Equation 7

The land value is a function of housing price, P, and the building cost, C, as an option value. To effect the risk-neutral condition in real option analysis, the drift parameter μ must be transformed to (μ-$\lambda\sigma$) where λ is the risk price of the housing asset. We may then derive the differential expression of L, as Equation 8, where i is the risk free interest rate:

$$[\frac{\partial L}{\partial t} + (\mu(L, X) - \lambda\sigma)P \cdot \frac{\partial L}{\partial P} + 0.5\sigma^2 P^2 \cdot \frac{\partial^2 L}{\partial P^2}] \cdot dt + \sigma P \cdot \frac{\partial L}{\partial P} \cdot dB - iL = 0$$ Equation 8

Substituting Equation 5 into Equation 6, we can get the expression of μ as Equation 9:

$$\int_0^t \mu(L, X)dt = \ln \alpha s + \beta \ln E(L \mid \psi) + \sum_1^n \gamma_i \ln X_i$$ Equation 9

To solve Equation 8 and 9, some boundary conditions are necessary. In brief, Equation 10 effects the continuity of the solution and Equation 11 is the smooth condition:

$$P(0) = 0, \; L(h^*) = P - C$$ Equation 10

$$\partial L / \partial P \,|_{h^*} > 0, \; \partial^2 L / \partial P^2 \,|_{h^*} > 0$$ Equation 11

The parameter h^* is the hurdle ratio indicating the optimal price-cost condition for the development. As the result of the constant marginal building cost assumption, the optimal development scale should always reach the maximum planning limitation.

From Equation 8 to Equation 11, we can explicitly derive the solution expressions of the model, as Equations 12 and 13.Here we refer to the work of Williams (1990), Quigg (1993) and Grovenstein et al (2011), where they gave a explicit model frame for solving the option valuation of land as a form: $L(t) / C = A(P(t) / C)^{\omega} + k$. The solutions are in the form of conditional probability so as to indicate their equilibrium relationship. The parameter ω is a proxy of housing market volatility, which is a complex function of i, λ and σ[10]:

$$E(P \mid \psi) = [\alpha s \cdot (\prod_1^n X_i^{\gamma_i}) \cdot C^{\beta(1-\omega)} \cdot (\omega - 1)^{\beta(\omega-1)} \cdot \omega^{-\beta\omega}]^{1/(1-\beta\omega)}$$ Equation 12

[10] The detail expression of ω is $\{[0.25\sigma^4 + \lambda\sigma^3 + i\sigma^2 + i^2]^{0.5} + 0.5\sigma^2 - i\} / \sigma^2$.

$$E(L\,|\,\psi) = [\alpha s^{\omega} \cdot (\prod_1^n X_i^{\omega\gamma_i}) \cdot C^{1-\omega} \cdot (\omega-1)^{\omega-1} \cdot \omega^{-\omega}]^{1/(1-\beta\omega)} \qquad \text{Equation 13}$$

To further simplify the form of solution, let：

$$\Gamma = \alpha s \cdot (\prod_1^n X_i^{\gamma_i}) / \omega \qquad \text{Equation 14}$$

$$\Phi = (\omega - 1) / C \qquad \text{Equation 15}$$

where Γ can be called as the "external factor" representing the effect from outside the land development, or say outside the option exercise process, and Φ the "internal factor" representing the inner effect from the land development. Substituting Equation 14 and 15 into 12 and 13, we gain the simplified expression of the solution as Equation 16 and 17, which are the target functions of our following numerical analysis:

$$E(P\,|\,\psi) = \Gamma^{1/(1-\beta\omega)} \cdot \Phi^{\beta(\omega-1)/(1-\beta\omega)} \cdot \omega \qquad \text{Equation 16}$$

$$E(L\,|\,\psi) = \Gamma^{\omega/(1-\beta\omega)} \cdot \Phi^{(\omega-1)/(1-\beta\omega)} \qquad \text{Equation 17}$$

Relationship Between Equilibrium Model and Classic Model

Because of the parameter of the Land Finance System, β, the housing price and land price is combined together into the endogenous option-pricing model. If we let β=0, indicating there is no effect of the Land Finance System on the housing market, the expected housing price, Equation 16, will be transformed to Equation 18 as an absolutely exogenous variable, while the expected land price, Equation 17, will be transformed to Equation 19, retrograding backwards to the original classic option-pricing model.

$$E(P\,|\,\psi) = \Gamma \cdot \omega = \alpha s \cdot \prod_1^n X_i^{\gamma_i} \qquad \text{Equation 18}$$

$$E(L\,|\,\psi) = \Gamma^{\omega} \cdot \Phi^{(\omega-1)} = E(P\,|\,\psi)^{\omega} \cdot \omega^{-\omega} \cdot C^{1-\omega} \cdot (\omega-1)^{\omega-1} \qquad \text{Equation 19}$$

NUMERICAL ANALYSIS

Basic Range of Critical Parameters

The range of parameters is important in both theory and practice and may be identified through the basic relationship between housing price and land price. From Equation 5, we gain:

$$\frac{\partial E(P\,|\,\psi)}{\partial E(L\,|\,\psi)} = \beta \cdot E(L\,|\,\psi)^{\beta-1} \cdot (\alpha s \prod_1^n X_i^{\gamma_i})$$

Because it has been verified that the option price is positively affected by the housing price (Hull, 2009), we have:

$$\frac{\partial E(P\,|\,\psi)}{\partial E(L\,|\,\psi)} = [\frac{\partial E(L\,|\,\psi)}{\partial E(P\,|\,\psi)}]^{-1} \geq 0$$

As a result, the other items are all positive, so we know that $\beta \geq 0$.

Further, because the land price will never exceed the housing price in practice, from Equation 16 and Equation 17, we derive:

$$\frac{E(P\,|\,\psi)}{E(L\,|\,\psi)} = \frac{\Gamma^{1/(1-\beta\omega)}\cdot\Phi^{\beta(\omega-1)/(1-\beta\omega)}\cdot\omega}{\Gamma^{\omega/(1-\beta\omega)}\cdot\Phi^{(\omega-1)/(1-\beta\omega)}} = \Gamma^{-\omega}\cdot\Phi^{\beta}\cdot\omega > 1$$

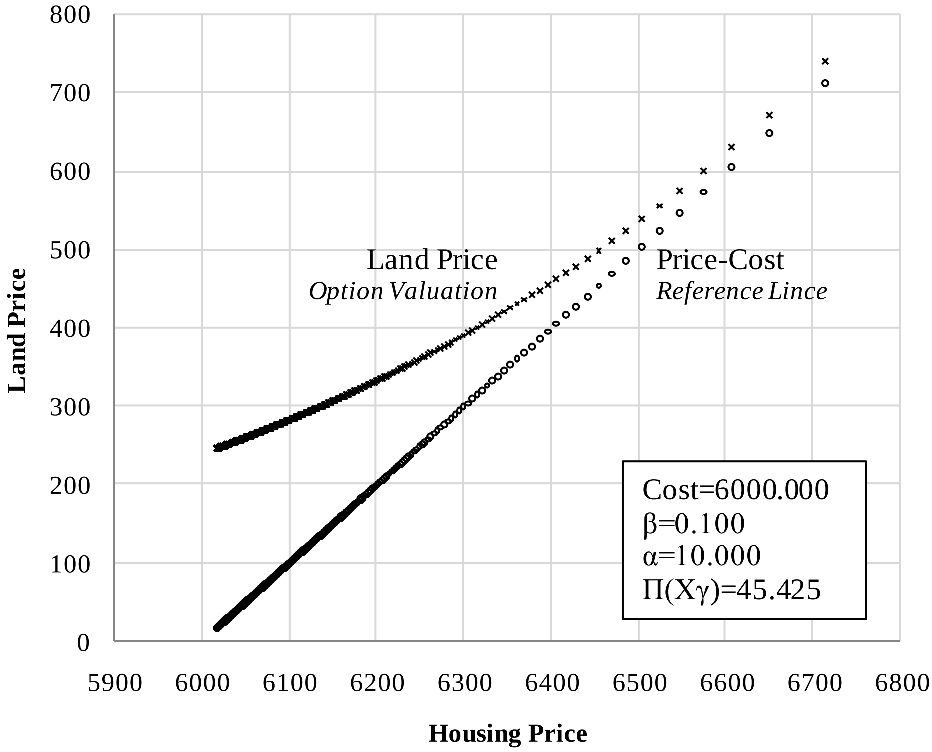
It is clear that Γ represents the exogenous characteristics of housing price, with the original exogenous housing price providing β=0, so it will always be of the positive value. We can therefore derive:

$$\begin{aligned}
&\Phi^{\beta}\cdot\omega > \Gamma^{\omega}\\
&\Rightarrow [(\omega-1)/C]^{\beta(\omega-1)}\cdot\omega^{\omega+1} > \alpha s^{\omega}\cdot\left(\prod\nolimits_1^n X_i^{\gamma_i}\right)^{\omega} > 0\\
&\Rightarrow \omega > 1
\end{aligned}$$

So far, we have estimated the basic boundary of the parameters' value. Based on this result, some general simulations can be undertaken to investigate the details.

Option Pattern of the Model

Similar to an option underlying a financial asset, the land price and the housing price should have an option-type correspondence. Based on our endogenous option-pricing model, the housing price is never completely exogenous, and the determinants that affect the housing price affect the land price too. It is to say, given the exogenous variables and the parameters, the housing price and the land price are simultaneously determined.



Simulation of Option Pattern for Endogenous Option-Pricing Model
Source: Authors
Figure 2

Figure 2 shows the simulation graphic of the correspondence pattern between housing price and land price calculated from the endogenous option-pricing model. In terms of the simulating condition, building cost is 6000 RMB yuan per square metre, β=0.100, αs=10.000 and $\prod X^{\gamma}$is supposed to be 45.425. In order to get a series of pricing pairs of the housing price and land price, the parameter ω is allowed to change with the step 0.01. From the graphic, we can see the typical pattern of the real option value that moves across the trigger point of strike price (6000 RMB yuan). The model clearly reflects the value gap between the land price and 0 at the trigger point, which demonstrates the value of uncertainty of the housing price and therefore the worth of the option-pricing model. With housing price going higher, the land price gradually converges towards the reference line, which is a 45-degree line reflecting the gap between housing price and the building cost.

Effect of the Land Finance System on the Markets

To investigate the effect of the Land Finance System on the housing price and land price, the first and second order partial derivatives of the expected housing price and land price are calculated, as shown in Equations 20 to 23, where $Z_\beta = \Gamma^{\omega} \cdot \Phi^{\omega-1} > 0$:

$$\frac{\partial \ln E(P \mid \psi)}{\partial \beta} = \frac{1}{(1-\beta\omega)^2} \ln(Z_\beta)$$

Equation 20

$$\frac{\partial \ln E(L \mid \psi)}{\partial \beta} = \frac{\omega}{(1-\beta\omega)^2} \ln(Z_\beta)$$

Equation 21

$$\frac{\partial^2 \ln E(P \mid \psi)}{\partial \beta^2} = \frac{2\omega}{(1-\beta\omega)^3} \ln(Z_\beta)$$

Equation 22

$$\frac{\partial^2 \ln E(L \mid \psi)}{\partial \beta^2} = \frac{2\omega^2}{(1-\beta\omega)^3} \ln(Z_\beta)$$

Equation 23

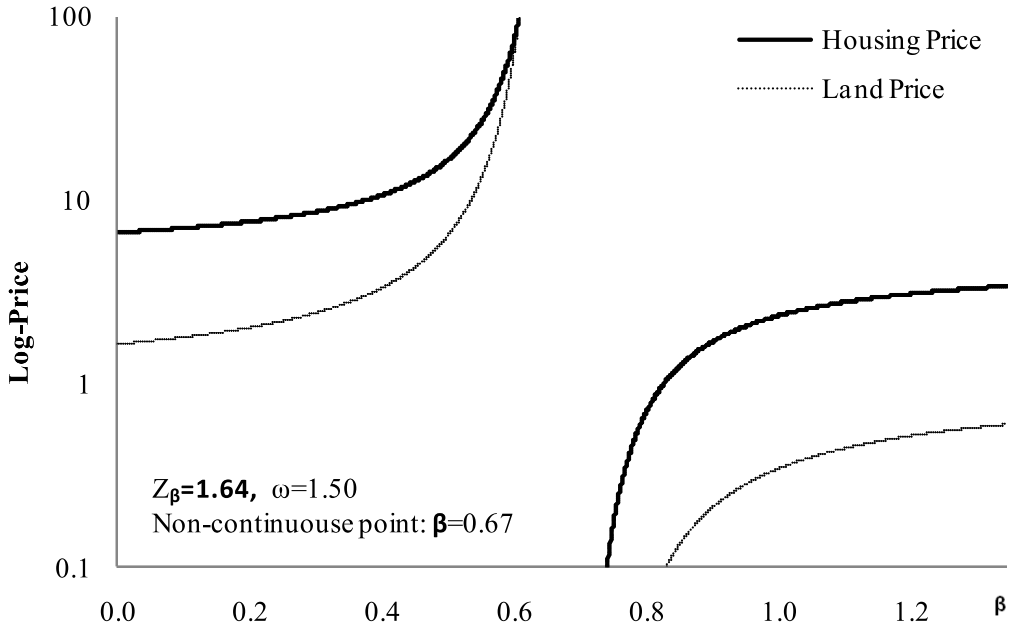
From Equations 20 to 23, we can find that both the expected prices are not continuous at the point $\beta=1/\omega$. The effect of β on the prices depends on both the intervals of β and Z_β, as shown in Table 3.

Z_β \ β	$0 < Z_\beta < 1$	$Z_\beta = 1$	$Z_\beta > 1$
$\beta \in (0, 1/\omega)$	Accelerated decrease*	Constant	Accelerated increase
$\beta \in (1/\omega, +\infty)$	Decelerated decrease	Constant	Decelerated increase

*Describe the movement of housing and land prices with β.

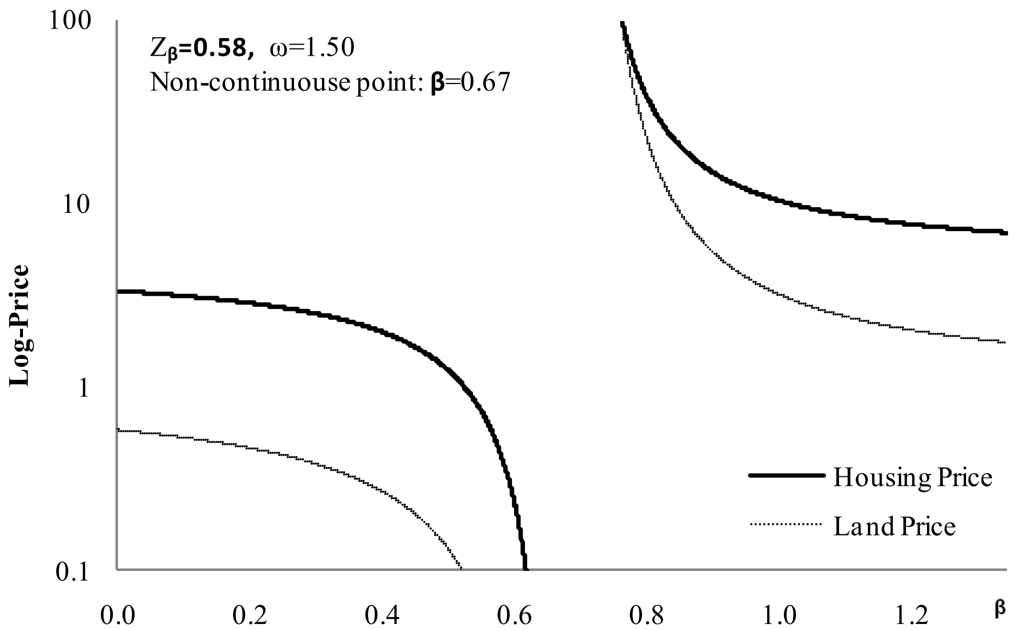
Effect of β on Option Valuation Under Different Condition
Source: Authors
Table 3

Figure 3 reflects these variations directly, of which Figure 3.1 seems to be a "good" situation, which is much closer to the situation over past years in China, where the facilities investment from land transfer income could both positively affect the housing and land price as the result of very strong demand in the housing market due to a rapid growing city.



Movement of Housing and Land Prices with β
Source: Authors
Figure 3(1)

Figure 3.2, however, might reflect a "bad" situation to an extent attributable to a declining duration and weak demand in the housing market. A "bad" market situation is inferable from a relatively lower Z_β in the model. There seems to be simultaneously two reasons that could lead to the very low value of Z_β, namely the extreme situation of the market, one is the expected weak price of the housing market and the other is the extremely high development cost.



Movement of Housing and Land Prices with β
Source: Authors
Figure 3(2)

Another interesting thing is the value of β itself, which can be treated as the elasticity of the percent change of housing price to the percent change of land value. Here, however, it is better to judge the level of the elasticity by the inverse value of the housing market volatility, ω, rather than the unit value,1, because no matter under a "good" or a "bad" situation the variations of the housing price and land value are very different according to the non-continuous point of β. Considering the continuous variation in society and economics in the real world and, meanwhile, the boundary condition that β equals 0 for the basic line, we believe that the situation of β less than $1/\omega$ coincides with reality even though the mathematical results show us many potentially different situations.

Regarding such situations where $\beta<1/\omega$, both the "good" and "bad" markets give us an intuition that β plays an interesting role in the variation for both the housing price and land value under the Land Finance System. In a "good" market, the looping interaction between housing price and land value can be explained as the following process with other things being equal:

1. a positive change in market-wide average housing price will raise the land value;
2. the increasing land value promises a larger amount of latent facilities construction funds;
3. a higher β realizes a larger investment in urban facilities;
4. the demand in a good housing market is strong enough to price the improved facilities fairly and the appreciation is added into the housing price around the location where the facilities are fostered; and
5. the market-wide average housing price will further increase due to such local appreciation of the housing price.

As the result of the fifth step, the market average housing price is again raised and the loop will go on till reaching the equilibrium point according to a given β as shown in Figure 3(1).

The process for a "bad" market is similar except for some inverse effects. The most different point is the fourth step, where the demand in a "bad" market is too weak to support and recognize the appreciation of the improved location value of the housing so that the overall change is still down even though there is a slight rise in the housing price due to such an improvement. As the result, only the shaded cells in Table 3 will occur in reality and the variation direction depends on the value of Z_β.

Effect of Volatility on the Markets

Similar to the analysis of β, we further investigate the effect of ω on housing price and land value. The first and second order partial derivatives of both the prices are expressed as Equations 26 to 29, where $Z_\omega = \Gamma \cdot \Phi^{(1-\beta)} > 0$ is defined:

$$\frac{\partial \ln E(P|\psi)}{\partial \omega} = \frac{\beta}{(1-\beta\omega)^2} \cdot \ln Z_\omega$$

Equation 26

$$\frac{\partial \ln E(L|\psi)}{\partial \omega} = \frac{1}{(1-\beta\omega)^2} \cdot \ln Z_\omega$$

Equation 27

$$\frac{\partial^2 \ln E(P|\psi)}{\partial \omega^2} = \frac{\beta}{(1-\beta\omega)}\left[\frac{1}{\omega(\omega-1)} + \frac{2\beta \ln Z_\omega}{(1-\beta\omega)^2}\right]$$

Equation 28

$$\frac{\partial^2 \ln E(L|\psi)}{\partial \omega^2} = \frac{1}{(1-\beta\omega)}\left[\frac{1}{\omega(\omega-1)} + \frac{2\beta \ln Z_\omega}{(1-\beta\omega)^2}\right]$$

Equation 29

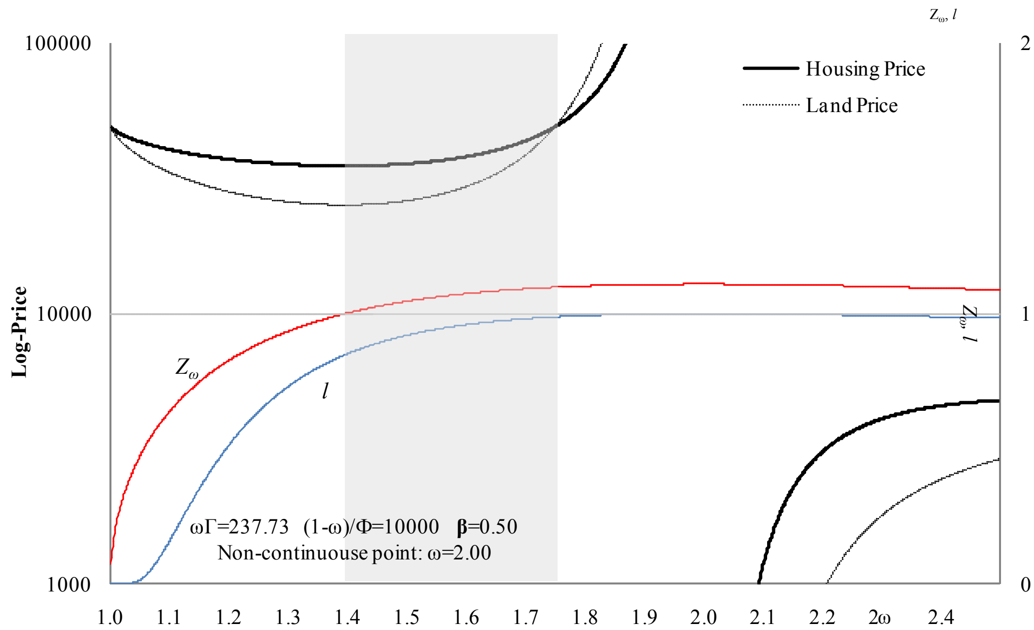
The expressions of the second order partial derivatives of ω are a little more complex than that of β that the interval [0,1] of Z_ω is further divided by a cut-off point, l, with the definition $l = e^{-(1-\beta\omega)^2/2\beta\omega(\omega-1)} < 1$, shown as Table 4. There is also a non-continuous point, $\omega=1/\beta$, for the variation of average housing price and land value with ω.

Z_β \ ω	$0 < Z_\omega < l$	$l < Z_\omega < 1$	$Z_\omega = 1$	$Z_\omega > 1$
$\omega \in (1, 1/\beta)$ *	Accelerated decrease**	Decelerated decrease	Constant	Accelerated increase
$\omega \in (1/\beta, +\infty)$	Decelerated decrease	Accelerated decrease	Constant	Decelerated increase

*Only if β<1. **Describe the movement of housing and land prices with ω.

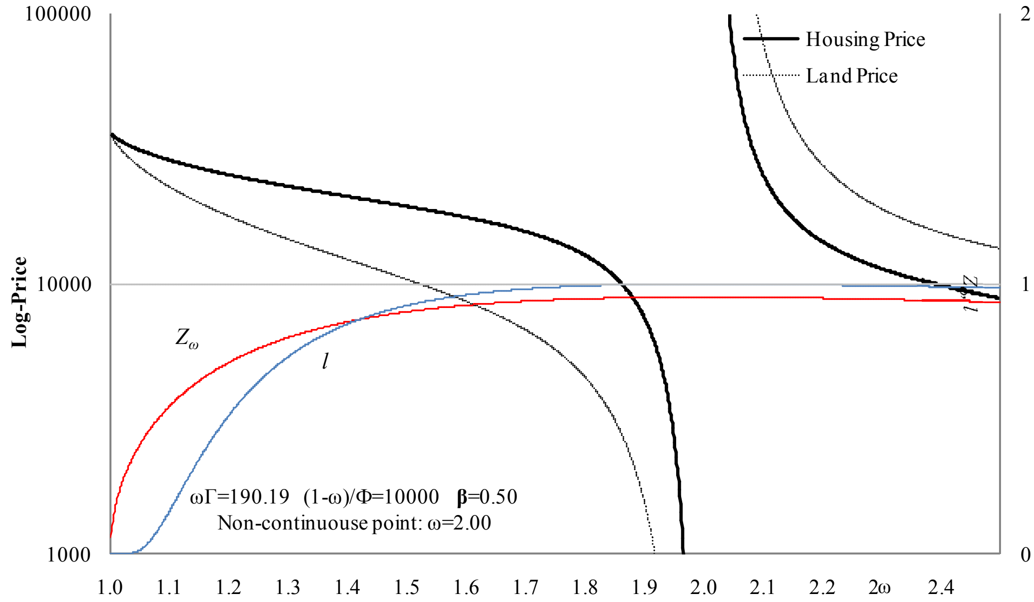
Effect of ω on Option Valuation Under Different Condition
Source: Authors
Table 4

The simulation results of the variation are shown as Figure 4, where Z_ω and l also change with ω, which results in the different conditions under Z_ω appearing in the same simulation graphic. Although any situation can be simulated as both the graphics in Figure 4 show, such situations that accord with the normal reality of the housing and land market are limited.



Movement of Housing and Land Prices With ω
Source: Authors
Figure 4(1)

As in Figure 4.1, where Z_ω is more than 1, the situation is much closer to the normal market especially within the interval of the grey area.



Movement of Housing and Land Prices With ω
Source: Authors
Figure 4(2)

However, the situation in Figure 4.2 will never happen in the normal market because here the land value is always negative according to the market volatility. However, it might possibly be assumed to occur in extreme situations of the market such as the Great Depression, where people are so sensitive to market risk that even a little more increase in volatility might sharply reduce the expected price of land and housing.

The sensitivity of the land value to the volatility of the average housing price is an essential characteristic of the real option. Although the exogeneity is impaired to an extent under China's circumstance, this core attribute of the real option never weakens. Nevertheless, there is also an interesting change that the housing price itself goes so far as to depend on its volatility. With ω increasing, in the "normal" interval in Figure 4(1), market-wide housing price also inclines endogenously. This simulation result reveals that the change of housing price volatility will always render a new equilibrium position between housing price and land value as shown in Figure 4(1) and Figure 4(2).

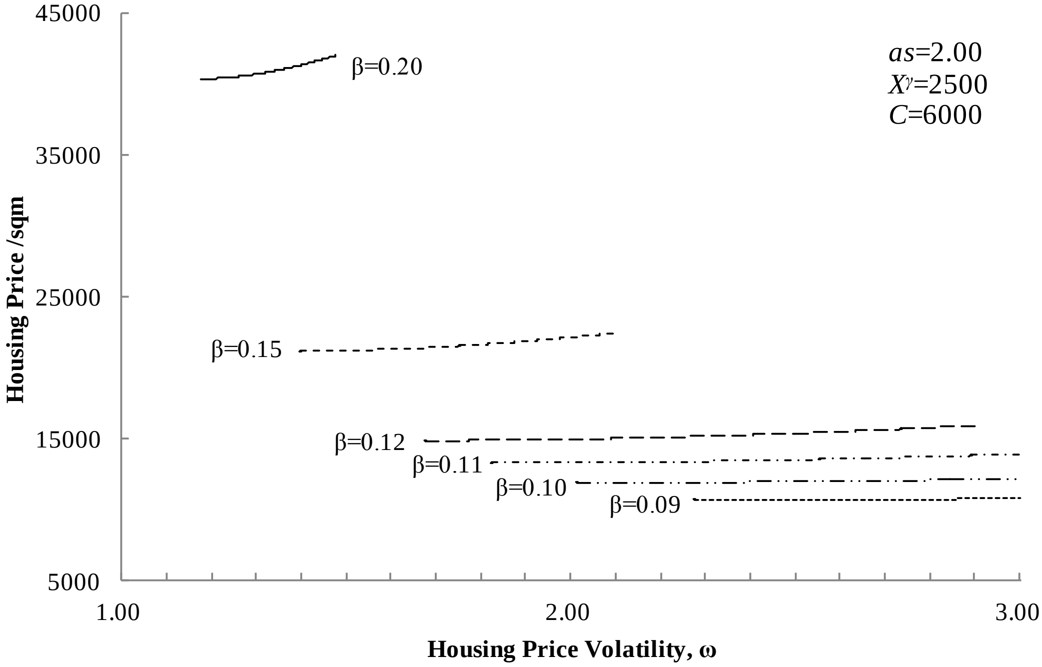
Because $\beta<1/\omega$ as we discussed above, we can understand that $\omega<1/\beta$ is a situation coinciding with reality. Furthermore, because of the general nature of real option theory with the volatility of underlying asset price attributable to the appreciation of the real option value, we can also infer that only the situation in Figure 4(1) might happen in a normal market while the situation in Figure 4(2) probably occurs only in a very extreme condition as discussed above. As a result, only the shaded cell in Table 4 will happen in reality, where Z_ω should always be more than 1 so that it is unnecessary to compute the complicated cut-off point l in practice.

A View on the Housing Boom in China

Based on the above discussion, an equilibrium real option model might provide us with a new tool to explain the housing boom in China under the Land Finance System, at least theoretically.

In summary, under the Land Finance System, the housing price volatility will influence the housing price itself as well as influencing the land value classically following real option theory. Actually, such effect appears via a looping interaction between market average housing price and land value triggered by the change of volatility and ending with a new equilibrium position for the price and the value. However, the degree of such looping or, namely, the final position of the new equilibrium depends on the system effect parameter, β.

Figure 5 shows a simulation result of the effect of market average housing price volatility on the housing price under different values of β. The curve of each scenario has different effective length reflecting the "normal" market conditions involving "Z_ω>1" and "land value will be less than the housing price".



System Effect on Housing Boom
Source: Authors
Figure 5

Figure 5 reveals much more clearly the system effect on the housing boom in a normal "good" market. There are two types of premium rendered by the Land Finance System. Firstly, along any single curve, the market average housing price inclines with the rising volatility of the market housing price because of the Land Finance System. So the first type of premium here is the risk-premium: the price risk of the housing market is added into the housing price itself due to the system. Obviously, the larger the β is, the bigger the risk-premium is. The upward degree of the curve with β=0.20 is much more than that with β=0.10. Therefore, the risk-premium is not only determined by the market volatility but also depends on the parameter β. The other type of premium could be referred to as β-premium, which reflects the gaps between curves in Figure 5. The β-premium is totally attributed to the value of β, which determines the looping interaction between housing price and land value under the Land Finance System. If β equalled 0, we would have only one horizon curve being independent with the housing price volatility.

CONCLUSION

This paper expands the real option pricing model to consider vacant land valuation within the specific system circumstance in China and, as a part of the results via simulations, provides some interesting findings to explain the housing boom over the past decade in China in terms of the specific system circumstance.

By adjusting the exogenous assumption of the underlying asset price from the classic real option pricing theory, we establish an equilibrium option pricing model under the condition of the Land Finance System in China. The model simultaneously determines the valuation of the target to-be-developed vacant land and the level of market-wide average housing price. There are two critical parameters in this model. One is the price volatility of the housing market, ω, which is also a very important factor in the classic real option model. The other one is the elasticity of the housing price on the land value, β, which arises from the Land Finance System in China. Through numerical analysis, we confirm that the valuation of land and the underlying housing price from the model are consistent with the typical nature of real options. Furthermore, we find the variation principles of housing price and land value with β and ω depend on the intervals of β and ω and some other internal and external conditions that may be represented by the calculating indicators Z_β and Z_ω.

The findings of this paper may be helpful to explain the housing boom in China over the past decade. After mid-2004, when, as the beginning, urban land was regulated to be transferred to any private developer only by the local government and the land transfer income to be gained only by the local government, the land value and the urban average housing price started to interact gradually and then the boom caused by this system began to grow. On one hand, the Land Finance System directly results in a housing price rise in equilibrium, because the investment amount on urban facilities founded from the land transfer income will increase the housing price by different extents. On the other hand, the Land Finance System makes a "bridge" between the housing price and its volatility. A higher market risk will boost the housing price itself because of the Land Finance System in China. This kind of premium may be very significant in an emerging market where a relatively large volatility in asset price always exists.

There are, however, a range of areas for further research, including the role of other macro-economic factors. We consider the building cost as a constant, which, although simplifying the expression of the model, sacrifices the accuracy of the theoretical proposition. Further, a log-linear housing price function was assumed in this paper, though there may be some other assumed basis with more solid foundation. Finally, although the numerical analysis has provided some meaningful findings, an empirical test is still needed, if a suitable sample set is available, both to investigate the correctness of the theoretical implications from the model and to estimate the value of β for different cities and therefore the value of the housing boom.

REFERENCES

Abel, AB, Dixit, AK, Eberly, JC and Pindyck, RS 1996, 'Options, the value of capital, and investment', *The Quarterly Journal of Economics*, 111, pp. 753-777

Aslund, O, Osth, J and Zenou, Y 2009, 'How important is access to jobs? Old question--improved answer', *Journal of Economic Geography*, 10(3), pp. 389-422

Bulan, L, Mayer, C and Somerville, CT 2009, 'Irreversible investment, real options, and competition: Evidence from real estate development', *Journal of Urban Economics*, 65, pp. 237-251

Capozza, D and Li, Y 1994, 'The intensity and timing of investment: the case of land', *American Economic Review,* 84, pp. 889-904
Capozza, DR and Helsley, RW 1990, 'The stochastic city', *Journal of Urban Economics,* 28, pp.187-203
Cheshire, P and Sheppard, S 1995, 'On the price of land and the value of amenities', *Economica,* 62, pp. 247-267
Cunningham, CR 2006, 'House price uncertainty, timing of development, and vacant land prices: evidence for real options in Seattle', *Journal of Urban Economics,* 59, pp. 1-31
Ding, W, Zheng, S and Guo, X 2010, 'The value of access to jobs and amenities: evidence for new residential properties in Beijing', *Tsinghua Science and Technology,* 15, pp. 595-603
Fisher, LM, Pollakowski, HO and Zabel, J 2009, 'Amenity-based housing affordability indexes', *Real Estate Economics*, 37, pp. 705-746
Grovenstein, RA, Kau, JB and Munneke, HJ 2011, 'Development value: a real options approach using empirical data', *The Journal of Real Estate Finance and Economics,* 43(3), pp. 321-335
Gyourko, J, Kahn, M and Tracy, J 1999, 'Quality of life and environmental comparisons', *Handbook of Regional and Urban Economics,* 3, pp.1413-1454
Harchaoui, TM and Lasserre, P 2001, 'Testing the option value theory of irreversible investment', *International Economic Review,* 42, pp. 141-166
Heckman, JJ 1979, 'Sample selection bias as a specification error', *Econometrica: Journal of the Econometric Society,* pp. 153-161
Hess, DB and Almeida, TM 2007, 'Impact of proximity to light rail rapid transit on station-area property values in Buffalo, New York', *Urban Studies,* 44, pp. 1041-1068
Holland, A, Ott, S and Riddiough, T 2000, 'The role of uncertainty in investment: an examination of competing investment models using commercial real estate data', *Real Estate Economics,* 28, pp. 33-64
Hull, J 2009, *Options, futures and other derivatives,* Pearson Prentice Hall
Kim, J and Zhang, M 2005, 'Determining transit's impact on Seoul commercial land values: an application of spatial econometrics', *International Real Estate Review,* 8, pp. 1-26
Krychowski, C and Quélin, BV 2010, 'Real options and strategic investment decisions: can they be of use to scholars?', *Academy of Management Perspectives*, 24(2), pp. 65–78
McDonald, R and Siegel, D 1986, 'The value of waiting to invest', *The Quarterly Journal of Economics*, 101.
Quigg, L 1993, 'Empirical testing of real option-pricing models', *The Journal of Finance,* 48, pp. 621-640
Rochaa, K, Sallesb, L, Garciab, FAA 2007, 'Real estate and real options: a case study', *Emerging Markets Review,* 8(1), pp. 67-79
Rhys, H, Song, J and Jindrichovska, I 2002, 'The timing of real option exercise: some recent development', *The Engineering Economist*, 47, pp. 436-450
Sing, TF and Patel, K 2001, 'Evidence of irreversibility in the UK property market', *The Quarterly Review of Economics and Finance,* 41, pp. 313-334
Titman, S 1985, 'Urban land prices under uncertainty', *American Economic Review,* 75, pp. 505-514.
Trigeorgis, L 1996, *Real Options*, MIT Press, Cambridge
Williams, JT 1991, 'Real estate development as an option', *Journal of Real Estate Finance and Economics,* 4, pp. 191-208
Williams, JT 1993, 'Equilibrium and options on real assets', *Review of Financial Studies,* 6, pp. 825-850
Yamazaki, R 2001, 'Empirical testing of real option pricing models using land price index in Japan,' *Journal of Property Investment and Finance,* 19, pp. 53-72

Zheng, S and Kahn, ME 2008, ‘Land and residential property markets in a booming economy: new evidence from Beijing’, *Journal of Urban Economics*, 63, pp. 743-757

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Email contact: liuhy@tsinghua.edu.cn