UNCERTAINTY, FLEXIBILITY, VALUATION AND DESIGN:

HOW 21st CENTURY INFORMATION AND KNOWLEDGE CAN IMPROVE 21st CENTURY URBAN DEVELOPMENT – PART II OF II

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ABSTRACT

The 21st century presents humankind with perhaps its greatest challenge since our species almost went extinct some 70,000 years ago in Africa. A big part of meeting that challenge lies in how the urbanization of three billion additional people (equal to the entire world population in 1960) will be accomplished between now and mid-century, on top of necessary renewal and renovation of the earth's existing cities. China alone will urbanize 300 million more people between now and 2030, equal to the entire population of the U.S., the world's third most populous country, in just 20 years. This is development on a scale and pace that is an order of magnitude greater than the past century, in a world resource and climate environment that is near the breaking point, in a context of greater technological, financial, and economic uncertainty than ever before.

To meet this challenge will require that we use the best tools in our kit, including ones that have become available to us only in this new knowledge and information-based century. Technology got us here, and technology will be key to getting us through. In this paper we will review and synthesize two important methodological developments in our profession that can help infrastructure and real estate physical development (i.e., urban development) to be accomplished more effectively and efficiently in a world of uncertainty.

The first methodological development is the honing of real options theory and methodology for practical application to identify and evaluate sources of flexibility in the design and operation of capital projects. The second development is the marriage of digital data compilation of property transactions records with the honing of econometric analysis methodology to allow the practical quantification of real estate and infrastructure asset price dynamics.

We argue that this latter development provides the key input to the former development, enabling a much more complete and rigorous treatment of design and evaluation problems for urban development. We also argue that an engineering systems approach to option modelling is likely to find better traction in actual professional practice than the economic theoretical models that have dominated the academic literature. We provide a concrete example by applying the suggested approach to the Songdo New City development in Korea.

The result can be better informed design and valuation and more efficient urban development laced with greater flexibility to avoid the worst down-side outcomes and to take advantage of the best up-side opportunities, saving vital resources of capital, land, raw materials, and energy.

Editor's Note: Professors Geltner and de Neufville's substantial paper has been divided into two parts. The first part (Vol 18, No 3, pps 231-249) includes the introduction and consideration of economic real option models, engineering models and Monte Carlo simulation. The second part (Vol 18, No 3, pps 251-276) includes a consideration of quantifying uncertainty or volatility through real asset pricing data and indexing, an application to Songdo New City and conclusions. For ease of reference, the abstract and references have been included in both parts.

Keywords: real options, design, digital, uncertainty, development, Songdo New City

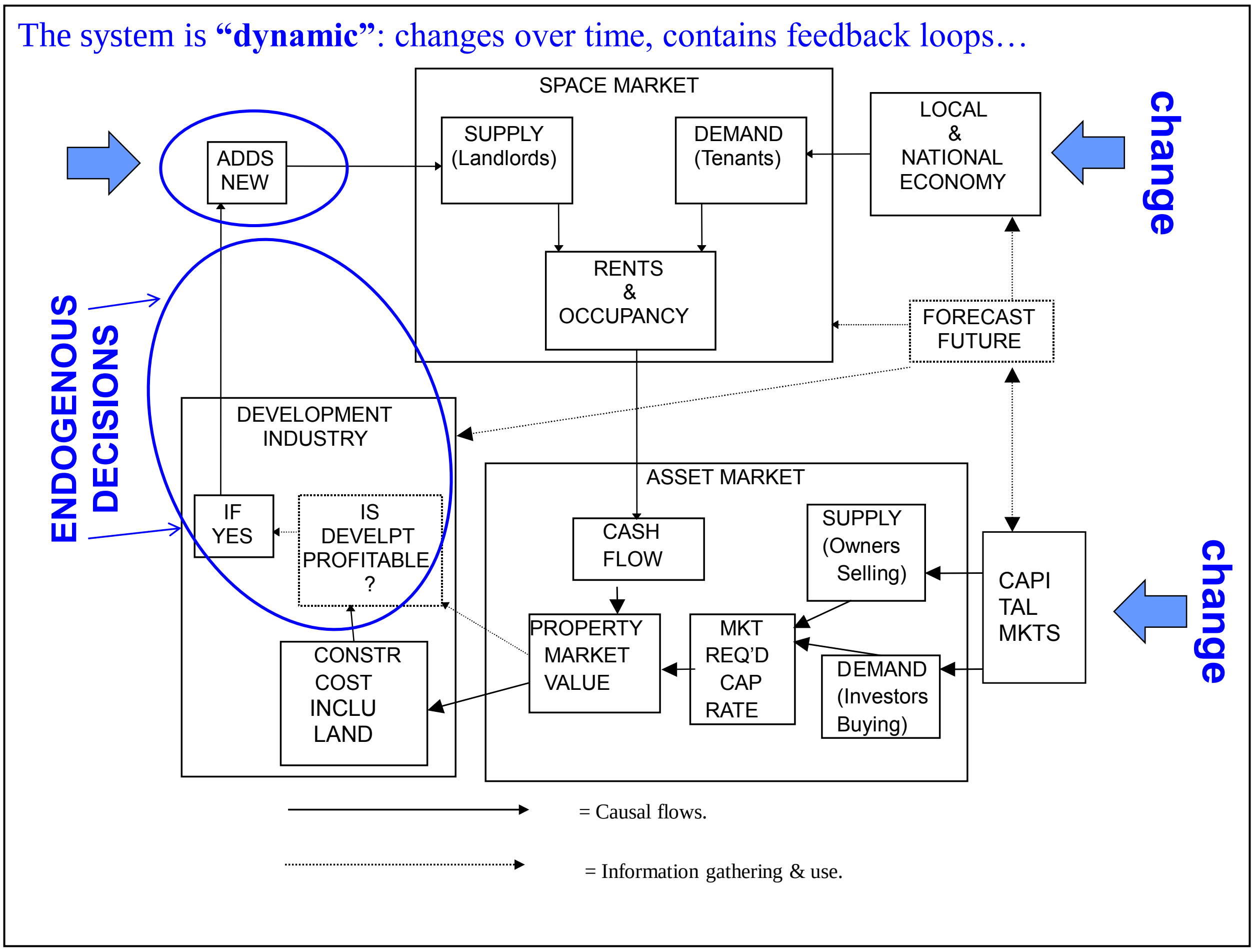
QUANTIFYING REAL ASSET UNCERTAINTY: PRIVATE MARKET TRANSACTION PRICE DATA AND INDICES

It is apparent that both the economics-based real option model and engineering-based simulation model are very sensitive to the quantitative assumptions about the magnitude of uncertainty as represented by the volatility in the underlying asset value. Yet until recently there has been a paucity of data to help quantify this vital input. As we have noted, a wealth of data on the historical returns to stocks, bonds, commodities, and foreign exchange has facilitated the practical application of option models in the finance industry. But most real estate assets trade, at the underlying level, in private search markets for whole (unique) assets rather than public auction markets for homogeneous shares or units. There has always been scepticism that one can simply assume that real estate assets have the same type and magnitude of volatility as stocks. And similar concerns have also applied regarding infrastructure assets, most of which have at least until recently not been privately owned or traded at all.

But recent decades have seen tremendous development in our ability to quantify the underlying volatility necessary for modelling the value of flexibility in real estate and infrastructure projects. Perhaps most fundamental has been the advent of rich datasets of property returns and price dynamics, at first based on appraisals and more recently based directly on transaction prices in the private property market. Also important, however, has been the blossoming and maturation of real estate and infrastructure sectors within the stock market, including most prominently the so-called “REITs” (Real Estate Investment Trusts), essentially “pure play” stocks, publicly traded but holding nothing but real estate investment assets. The infrastructure privatization movement that largely began in the 1980s has also enriched the record of traded infrastructure assets. In this section we will briefly review these developments as they bear upon the flexibility modelling discussed previously. We will sketch the major sources and types of uncertainty and volatility in real asset values over time.

Let us begin by stepping back and considering the fundamental sources of the uncertainty that underlie the volatility in asset values that we seek to quantify. A good way to do this is to envision the entire “system” within which real estate or infrastructure physical assets are embedded. Exhibit 4 presents a picture of the essentials of this “real asset system”. There are three major parts to the system: two different types of markets, and a productive industry that links them.

The large encompassing box at the top of the exhibit is the “space market”. This is the most fundamental part of the system, its *raison d’être*. This is the market for the <u>use</u> of the built space or facility. On the demand side of this market are potential tenants or other types of users of the buildings and infrastructure assets (travellers using bridges, shippers using ports, shoppers using retail stores, dwellers using apartment flats, and so forth). It is fundamentally by serving such demand, and <u>*only*</u> by that service, that real assets add value and make a positive contribution to the economy and society in which they are located, and this is true whether the assets are owned in the private or public sector. Underlying the demand side of the space market is the local and national (or even global) economy, which determines the amount, type, and location of spatially-fixed physical assets that will be useful and of value. On the supply side of the space market are landlords or other types of owners (public or private) controlling and managing the operation and usage of the built assets. The equilibrium between supply and demand in the space market determines the price of usage and the physical amount or rate of usage of the real assets (such as occupancy rate in buildings, or flow-through rate in transport infrastructure). The combination of this price (such as rent or tolls) and physical usage rate determines the magnitude of the benefit flows attributable to the real assets in the space market.[22] This annual benefit flow (a net income stream in the case of priced assets) is the output from the space market to the asset market.



The Real Asset System
Source: Authors
Exhibit 4

The asset market is where the ownership of real assets is traded. For our purposes it may also be thought of as where such assets are evaluated even if they are publicly owned and not traded. Most fundamentally, the asset market is where future streams of cash flows (or, more fundamentally, user benefit flows) are evaluated and (often) traded. Mechanically, the asset market converts such future streams into a present value. It does this by forecasting what the future benefit flow streams are expected to be, making a judgment about their risk, and discounting to a present certain value to reflect time and risk.

The asset market is a branch of the capital markets, where money and long-lived assets are traded. The capital market determines the opportunity cost of capital (OCC), including the price of time and risk. Even for public sector investments in assets that will not be traded, the capital for the investments comes ultimately from the capital markets, displaces private investment on the margin,

[22] In the case of asset usage that is not priced, such as roadways that are not tolled, one must quantify a social "shadow price" reflecting the marginal social benefit of the asset usage, in order to quantify the benefit flow from a public or social perspective.

and therefore faces the same OCC from a fundamental economic or social perspective.[23] The OCC relevant to each project serves as the discount rate for converting forecasted future cash flows into present value. As a shortcut pricing convention in the asset market, one often thinks of a simple "direct capitalization rate" (or "cap rate" in American real estate parlance), which is an initial annual net income yield rate, that one can use to quickly value assets by simply dividing their current annual net income by their cap rate. While the cap rate is just a stylized short cut, it can be thought of as reflecting the pricing in the asset market, reflecting the equilibrium between supply and demand of and for investment in capital assets of the type being traded in the asset market in question. As in the case of the space market, the underlying source of this supply and demand is at least partly exogenous to the asset market itself, reflecting the broader global capital and money markets, which in turn are products and parts of the world economy and information system.

In the real asset system depicted in Exhibit 4 we view the output of the asset market as being the prices (values) of existing (or potential proposed) real assets, including also the price of land sites necessary for the construction of spatially fixed real assets. These asset prices are a key input into the third major element in the overall system, which is the physical development industry, which governs the construction of new real assets. The development industry takes financial capital (money) from the capital markets and converts it into physical capital (spatially-fixed real assets) that add to the stock of supply in the space market, thereby completing the central loop in the overall system.

This central loop is characterized by a negative feedback mechanism that acts to help keep the entire system in balance, a bit like a thermostat in a heating/cooling system. The negative feedback occurs as the development industry makes a crucial comparison: between on the one hand the expected value of the benefits of the development represented by the price the proposed new real assets could fetch in the asset market, and on the other hand the costs of the development (including the opportunity cost of land and the necessary profit expectation for the developers and their financial backers). Only if and when the benefit exceeds the cost (i.e., there appears to be sufficient profit) will development be undertaken. Thus, if supply overshoots demand in the space market, this will tend to depress prices (rents) and/or average usage (occupancy), which will reduce the benefit (income) flows achievable by the real assets, which will in turn reduce their valuations (prices) in the asset market (holding cap rates constant), thereby tending to make the benefit/cost comparison in the development industry come up less favourable, resulting in a cut off or cutback in the supply of new real asset stock into the space market.

Note however, that this negative feedback loop is not perfect. It is a bit of a "ratchet", in that real assets tend to be very long-lived. Once built, they are "out there" for a long time, even if demand for their usage declines. With a well-functioning capital market and development industry the physical supply of real assets increases much more easily or quickly than it decreases.

Exhibit 4 highlights the sources of uncertainty in the real asset system we have just described. In the first instance, uncertainty exists in the two markets. What will be the prices and usage levels and therefore the magnitude of income or benefit flows coming out of the real assets in the space market? What will be the future expectations and the relevant interest rates and OCC and cap rates applied to the current and expected future benefit flows in the valuation of the real assets in the asset market? In both these cases, a fundamental source of the uncertainty is exogenous to the real asset system itself. As indicated in the exhibit, it is ***change*** in the (largely external) sources of usage demand and of capital supply and demand (including the forecasting of relevant future scenarios by

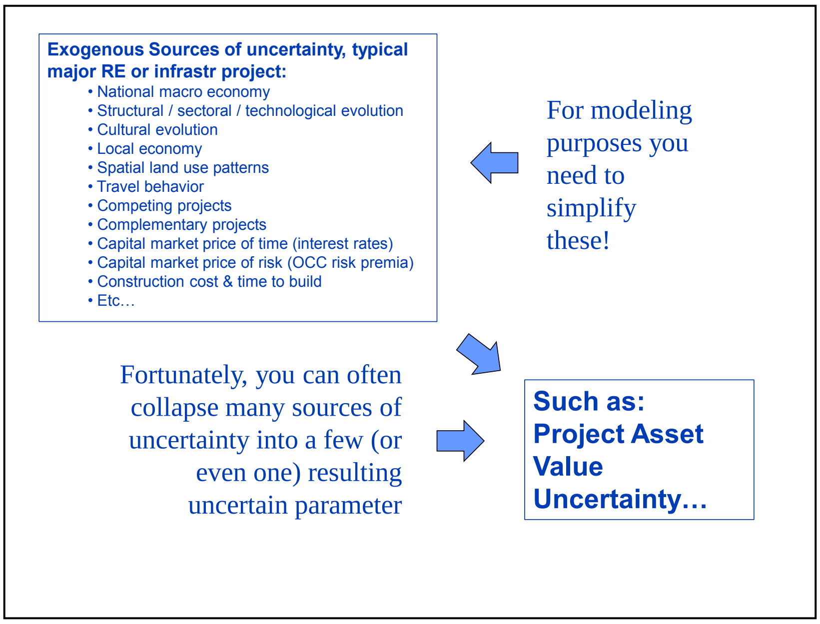
[23] Keep in mind that the OCC reflects the risk and cash (or benefit) flow timing for each specific project. For public entities the social or economic OCC may differ from the accounting cost of capital that may be what matters from a budgetary perspective.

players in those external systems) that governs the demand in the space market and the pricing and capital flow in the asset market.

But there is also an internal source of uncertainty within the real asset system itself, because the system is dynamic and contains the development industry that produces the physical assets that compose the system. We have noted the negative feedback loop that tends to keep the system in balance, but we also noted that this loop is not perfect. It is not perfect in part because no one can have a “crystal ball”, and we have noted the external sources of change acting on the other parts of the system, change that cannot be perfectly foreseen. It is also not perfect because of the aforementioned “ratchet” or asymmetry between additions to, versus subtractions from, the stock of real assets (long-lived assets once built do not go away).

But there is another source of imperfection in the system’s negative feedback loop, and this is a source that has only received serious attention among economists in recent decades. Here we are speaking of human behavioural characteristics that may cause a tendency toward certain types of systematic or persistent errors in economic and business decision-making. For example, people may tend to be overly optimistic; they may tend to feel more confident than they actually should be about their knowledge and their decisions. They may over-rate small probabilities and under-rate larger probabilities of negative events. They may also exhibit “herding” or “contagion” type behaviours, and be overly conservative in some ways, irrationally shying away from or delaying admission of losses.

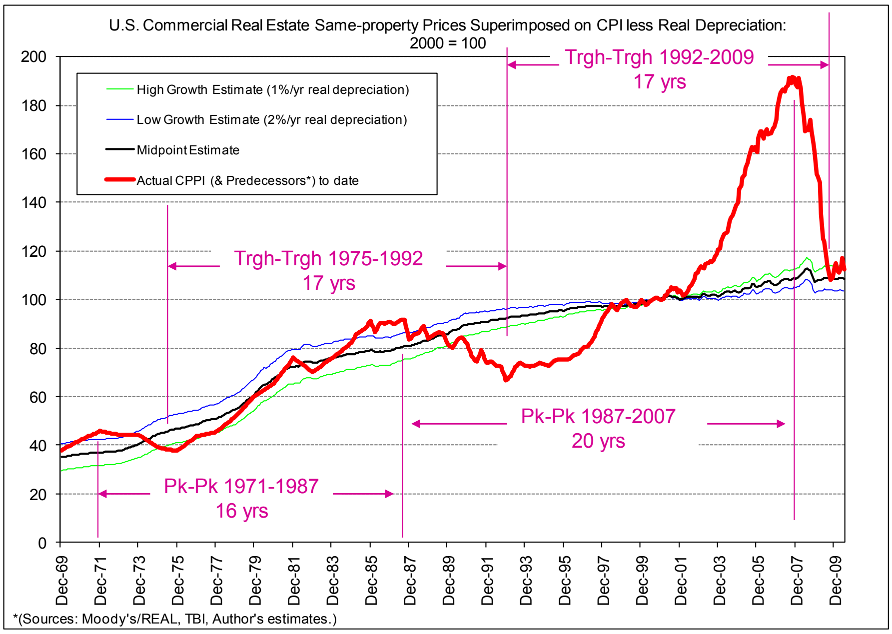
Furthermore, these behavioural phenomena are quite widespread. It would appear that in the real asset system they operate not only in the development industry but also in both of the other two main parts of the system, the space market and asset market. One result can be positive feedback loops that can occur in either the space market or the asset market. An upsurge in prices (caused perhaps by an exogenous shock) can cause a herd to push up prices further, potentially causing the price rise itself to fuel a further increase. Clearly bubbles can form and burst in asset markets, and also in some types of space markets.



Modelling Uncertainty in Real Asset Development Projects
Source: Authors
Exhibit 5

Thus, there are both exogenous and endogenous sources of uncertainty that cause volatility in real asset prices and valuations. And these sources are many and difficult to predict. Exhibit 5 presents a "laundry list" of typical sources of uncertainty (and some of these should be viewed as being possibly either exogenous or endogenous, with the list being only representative not exhaustive). But the point of Exhibit 5 is that, while one should certainly consider specific and underlying sources of likely future change, it is often possible for practical modelling purposes to collapse all such fundamental sources into one or a few key resulting variables. In particular, the most all-encompassing and often most useful such variable is the volatility in the real asset value, that is, the longitudinal variation in the price change (or value change) of the asset being developed. This volatility will typically reflect and be caused by all or most of the underlying specific sources of uncertainty. The lesson in the laundry list in the upper-left of Exhibit 5 (and in our prior discussion in this section) is to be humble, and not be surprised if the real asset volatility is substantially higher than you might have initially supposed!

With the above as background, let us consider what the recent databases and econometric discoveries can tell us about the nature and magnitude of uncertainty and volatility in real asset values relevant for quantifying models of the value of flexibility in development projects. As noted, a central challenge has been to put together a good empirical history of asset prices in the private property market. Thus, a good place to begin is with the big picture of the history of U.S. commercial real estate prices for "major assets" (aka "institutional property", generally large assets held by professional investment institutions such a pension funds). A picture of this history spanning four decades is shown in Exhibit 6, highlighting the long-term secular trend in (same-property) prices, as well as the three major asset-pricing cycles that have occurred during that history.



The Secular Trend and Long Cycle in U.S. Commercial Property Major Assets
Source: Authors
Exhibit 6

The first serious attempt to track investment property prices was the development of the NCREIF Property Index (NPI) in 1982. This index is based on appraised values rather than on transactions prices, which raises some problems, but it was a path-breaking and important starting point, and researchers have developed ways to "unsmooth" or "de-lag" such appraisal-based indices to improve their accuracy (Geltner et al 2003). More recently, the first regularly published transactions based index of commercial property in the U.S. was developed at the MIT Center for Real Estate, based on the NCREIF properties that sell out of the index. This so-called "TBI" (for Transactions Based Index) has been published since 2006 but its history goes back to 1984 at the quarterly frequency (see Fisher et al 2007).

Another index developed in 2006 is the Moody's/REAL Commercial Property Price Index (CPPI), which has been a monthly-frequency index beginning in 2001 reflecting a much broader and larger dataset of property transactions based on the Real Capital Analytics Inc (RCA) transactions database. (This database is still limited to the "institutional" or "major asset" category of properties, that is, not reflecting small "mom-and-pop" properties that number in the millions in the U.S. but amount to only a fraction of the dollar volume of sales or value of real estate assets). The Moody's index is a *repeat-sales* index, a methodology popularized by Case and Shiller (1987) based on the percentage change in the prices of the same properties sold more than once at different points in time. This type of index is very analogous to a stock market price index, in that it reflects the actual round-trip investment experiences of investors buying and selling properties in the asset market.

Exhibit 6 reflects a splicing together of these three indices (and some predecessor funds), to provide a reasonably fine-grained and consistent history since 1969.[24] While 40 years is less than we would like, and less than what is available for stocks and bonds, it is longer than most investment and construction project planning horizons and long enough to give a meaningful picture of the overall typical price dynamics of commercial real estate assets in the private property market.[25] Moreover, the quantity and quality of the transactions price data has greatly improved in recent years, along with the capability of econometric methods for digesting such data and producing accurate price indices. This is what has permitted the development of the TBI and Moody's indices.

There are two striking features of the price history in Exhibit 6. First is the long-run secular trend. This appears to be slightly less than the rate of inflation. The black line in Exhibit 6 traces a path that is 1.5% per year *less* than the U.S. Consumer Price Index (CPI). This line appears to represent a long-run sustainable price trajectory for same-property (aging) assets. Of course, this reflects the average context for major commercial property assets in the United States, a land-rich country where the historical period in question witnessed major improvements in transportation and telecommunications technology and infrastructure, resulting in a flattening of rent gradients and relative decline in location rent premiums in central places. The needs and preferences of real estate space users also evolved substantially over the historical period in question, as for example "Class A" office buildings that could get by with copper wiring and a neat lobby in the 1970s by the 2000s needed to include fibre-optic cable, atriums, physical fitness centers, natural lighting, and perhaps a heliport on the roof! The result was (as is typical) substantial depreciation in the value of the built

[24] The index in Exhibit 6 uses the "best" available index over each span of time, starting (in reverse order) with the monthly Moody's index since 2000, continuing back with the quarterly TBI through 1984, and then an annual-frequency version of unsmoothed/de-lagged NPI back to that index's inception in 1978, with similar predecessor series based on early institutional comingled funds prior to that. Throughout, the series is constructed to reflect "same property" price movements, that is, to include the effects of the aging and depreciation of existing structures, excluding major renovation/rehabilitation projects which require new capital infusion and therefore would obfuscate returns on pre-existing investments.

[25] There are a few *ad hoc* historical studies that provide much longer series, including a more-than-three-century history of annual price changes on the Herengracht canal in Amsterdam (see Eichholtz and Geltner 2002), and in general these longer histories seem broadly consistent with the type of dynamics displayed in Exhibit 6.

structure due to functional as well as physical and economic obsolescence. Thus, it is not surprising that the long-run secular trend in asset values reflects "real depreciation", price growth slightly less than inflation.

The second striking feature in Exhibit 6's pricing history is its strongly cyclical nature. The 40-year history traces three strong and clear asset pricing cycles, each one of notably similar length, with a period between 16 and 20 years (peaks in 1971, 1987, and 2007; and troughs in 1975, 1992, and 2009). Each of these cycles was quite dramatic at the time when it turned down, reflecting peak-to-trough declines in same-property real (inflation adjusted) average pricing on the order of 40% (as a fraction of the peak value). The persistence of this cyclicality is notable, particularly considering that the period of the cycle is shorter than the average real estate professional's personal career span and hence the upswings that lead to the downturns must occur in the presence of considerable institutional memory of the last crash. The cycles seem to be associated with increasing use of debt and a relaxation of lending standards. While each cycle is unique in terms of its specific historical causes and characteristics, there does seem to be a strong "rhyme and meter" in the commercial property market. Conflated with the cyclicality, there is an average annual volatility in the Exhibit 6 index on the order of 10%, somewhat less than the U.S. stock market's 15-20% long-run average annual volatility range (depending on what stocks one includes and what span of history one includes), and the property index displays considerable serial correlation or inertia (much more than stock market indices).[26]

While the type of aggregate or overall price dynamics portrayed in Exhibit 6 is very important for understanding the nature and magnitude of volatility in real assets, this level of analysis is not the only one that is important for modelling uncertainty in the valuation of individual assets or projects. Exhibit 7 depicts the dispersion of individual asset prices around the prices predicted by the regression model that underlies the transactions based index (TBI) noted earlier. In other words, the dispersion shown in the exhibit is *in addition* to the aggregate volatility represented by the index. It represents a type of transaction price "noise", reflecting the fact that it is difficult to precisely know the value of any individual asset at any given point in time, due to the uniqueness of each asset and the thinness of trading in the real property asset market. As a result, in any transaction price (or any valuation estimate) of an individual asset at a given point in time there will be random dispersion around some unobservable "true value" (or *ex ante* price expectation). This is a source of uncertainty that is akin to volatility, but that is non-temporal, that is, not substantially a function of the passage of time.[27] As shown in Exhibit 7, the indication from the TBI data is that this type of random dispersion may typically be on the order of +/-15% (standard deviation) around the average or expected price.[28]

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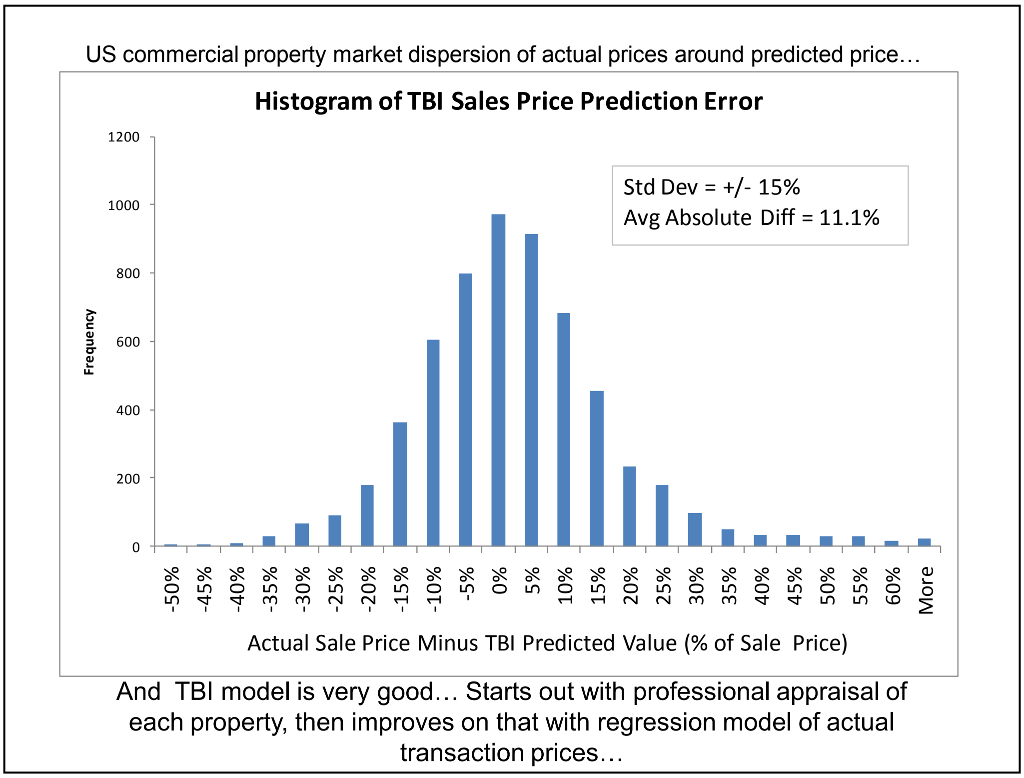
[26] Both the volatility and the autocorrelation were considerably greater during the most recent decade, 2000-2010. During that decade the annual calendar year volatility in the Moody's index was 15%. This of course reflects the great financial crisis of 2008-09.

[27] A slight exception to this point is the fact that a prior transaction price does in principle provide some information about the value of the asset in question, and thus, the longer it has been since the asset previously transacted, the greater may be the price noise component depicted in Exhibit 7. This type of "heteroskedasticity" is explicitly accounted for in the Case-Shiller type of repeat-sales indices such as the Moody's index noted earlier.

[28] As we will note shortly, the 15% standard error figure is also supported by analysis of the larger RCA dataset underlying the Moody's index. However, there is reason to believe large scale "mass appraisal" type models such as those used to construct indices like the TBI and CPPI would not be as accurate as more specifically targeted valuation estimates for individual subject properties. Evidence from other studies suggests that valuation noise may be closer to a 10% standard error in more targeted analyses. (See for example Crosby et al 1998, and Diaz and Wolverton 1998.)

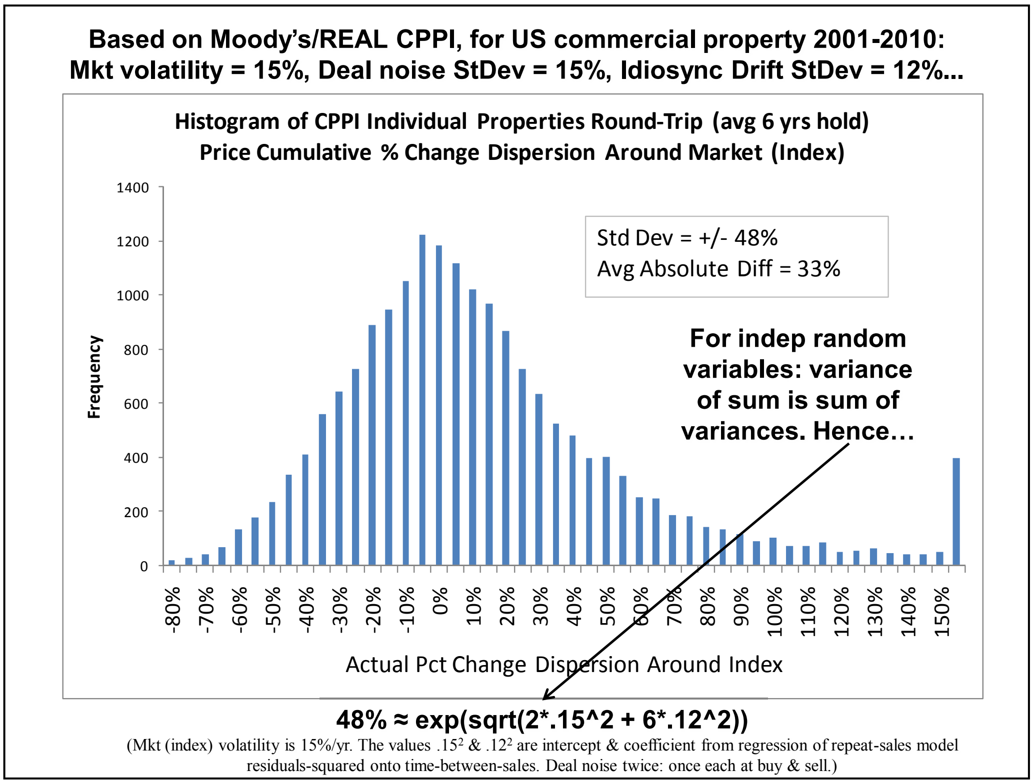


US Commercial Property Market Dispersion of Actual Prices Around Predicted Price
Source: Authors
Exhibit 7

In addition to the aggregate temporal volatility portrayed by indices, and the non-temporal valuation noise that occurs when (but only when) an asset is transacted or valued, another component of asset value volatility that is important in flexibility valuation is the temporal idiosyncratic "drift" (or "idiosyncratic risk" or "specific risk") that causes individual asset values to gradually diverge from aggregate indices or market average valuations. This reflects the fact that individual buildings or assets experience value evolution that is specific to themselves that is unrelated to the market as a whole, or is cancelled out in the aggregate average price evolution represented by the index. This source of volatility is captured in the residuals from the repeat-sales regressions that underlie indices such as the Moody's, and is depicted in Exhibit 8.

As noted in the exhibit, the dispersion in the residuals of a repeat-sales index reflects the combination of the non-temporal deal noise discussed previously as well as the temporally-accumulating idiosyncratic drift. By regressing the squared residuals onto a constant and the time between the two sales within each of the repeat-sale pairs (that make up the observations on which the repeat-sales regression is estimated), we can identify how much of the residual dispersion is due to each of these components. It turns out that the estimate of the magnitude of the non-temporal deal noise in the RCA database is very similar to that in the TBI residuals noted previously, approximately a 15% standard error (this despite the fact that the regression models and index methodologies are rather different, and the estimation database is different). The estimate of the rate of idiosyncratic risk or drift accumulation is approximately 12% per year (or more precisely, the variance accumulates at the rate of 12%-squared per year[29]).

[29] In other words, as noted in the Exhibit, the 48% standard deviation in the repeat-sales residuals from the regression model can be decomposed as the square root of the sum of twice the non-temporal dispersion whose variance is 15 percent-squared (once for each transaction in the transaction pair) plus six times the annual idiosyncratic drift variance



Histogram of CPPI Individual Properties Round-Trip (Avg 6 Yrs Hold)
Cumulative Percentage Price Change Dispersion Around Market (Index)
Source: Authors
Exhibit 8

We have now discussed and in some measure quantified five different types or sources of uncertainty or volatility in real asset values over time: the long-term secular trend rate in asset price growth, the long-run cyclicality in the asset market, the aggregate market-wide annual volatility exhibited in the market price index, the non-temporal price or valuation noise component, and the temporally accumulating idiosyncratic drift of individual property prices around the aggregate or market price evolution. But there is at least one additional source or type of uncertainty, and this is perhaps the most difficult to observe and measure empirically.

Asset valuations over time occasionally reflect relatively rare and difficult to predict major market-wide historical "jumps" or "black swan" type events, reflecting the "fat tails" of a non-Gaussian stochastic process. There is some reason to believe such "events" tend to occur more on the down side, value drops, rather than on the up side. Such an event is probably well represented in the U.S. commercial property asset market by the great financial crisis of 2008-09. The effect of such a "fat tail" event is graphically illustrated in Exhibit 9, which also serves to portray another important, and relatively recently matured source of data, individual REIT share price evolution histories from the stock market.[30]

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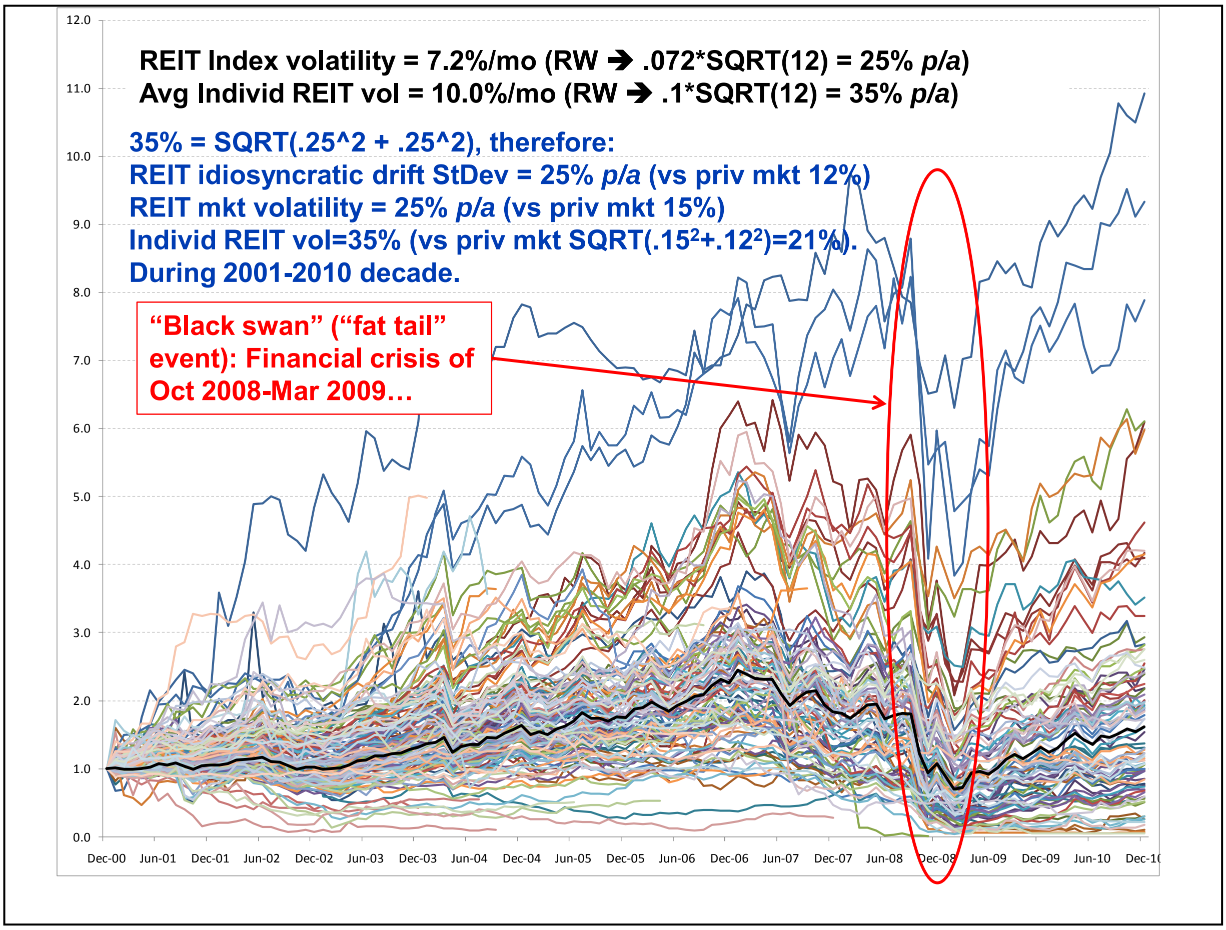
of 12 percent-squared per year (reflecting the average holding period between the repeat-sales of approximately six years).

[30] The U.S. REIT industry greatly expanded and matured during the 1990s, making it for the first time a robust and well followed sector of the stock market, as well as an important source of capital and active player in the private property market (as REITs buy, sell, and develop major properties in that market).



Individual Price Histories of 111 REIT Stocks From 2001 Through 2010
(Relative to Value as of December 2000 = 1.00,
Black Line is Unweighted Average Across All the REITs)
Source: Authors
Exhibit 9

Exhibit 9 depicts the price evolution of all 111 U.S. REITs that were publicly traded and tracked by the CompuStat database as of December 2000. The exhibit shows the evolution of each of these individual REIT's common stock share prices relative to what they started out at in December 2000, through December 2010. During that period, 47 REITs disappeared or dropped out of the database, so that only 63 REITs finished the decade. (The disappearances were due to de-listings and buy-outs, as well as to mergers and acquisitions. Very few if any REITs were delisted as a result of bankruptcy.)

As noted previously, REITs in the U.S. are relatively "pure plays" that are limited to investing in real estate assets. (They are not "merchant builders" who simply reflect the construction industry.) As such, REITs represent a way to understand the price dynamics and evolution of real estate asset values over time. While each individual REIT typically holds numerous individual properties, and this should reduce REIT volatility due to the diversification effect in the property portfolios, in fact REITs tend to specialize, and they also reflect management and "entity level" risk. The result is that the stock market tends to price individual REITs as single entities, as if each REIT is an individual asset in itself, albeit a somewhat different type of asset than an individual property.

Stock market valuation is a different type of process than private market asset valuation. Furthermore, REITs are typically levered, so their stock price movements may be magnified by the effect of their capital structure. For all these reasons, REIT share price dynamics represent a somewhat "different animal" compared to the private asset market valuations we have been analysing previously in this section and which are more directly relevant to the modelling of flexibility in development projects. Nevertheless, the stock market is a highly efficient and liquid asset trading arena characterized by very fast price discovery. And as REITs are essentially and fundamentally "real estate assets", it is relevant and enlightening to see what REIT valuation dynamics look like over time.

With this in mind, we note that the (unweighted) average annualized volatility across all the REITs in Exhibit 9 was 35%. This reflects the combination of aggregate REIT "market" risk plus individual REIT specific risk (or idiosyncratic volatility). Taking the average REIT return represented by the black line in the exhibit as representing the aggregate market (that is, the REIT market or REIT "sector" within the stock market), we note that this had an annualized volatility of 25%. This suggests that individual REIT idiosyncratic volatility also averaged about 25% (as $SQRT(0.25^2 + 0.25^2) = 0.35$). Thus, both in the aggregate, and regarding individual asset idiosyncratic returns, REITs appear to be more volatile than private real assets as traded directly in the private market. In part this may reflect the leverage in the REITs. In part it may reflect (and help to support) the different functioning of a public auction share market as compared to a private search whole-asset market, including the type of liquidity that public exchanges are designed to provide.[31]

Perhaps of more interest in our current context is the fact that the history of REIT share price movements during the 2000s decade depicted in Exhibit 9 presents a dramatic and graphic picture of the major "black swan" event of the 2008-09 financial crisis that we noted earlier. This is seen in the gigantic "downdraft" that pulled virtually all REITs down together and drastically during the October 2008-March 2009 period. During that period, REITs prices plunged farther than they ever had in the history of the industry, and they did so all together almost in unison, in effect, idiosyncratic drift was temporarily cast aside as all assets moved as one. There then occurred a quick and strong recovery, as the stock market apparently perceived that as regards most REITs, Mark Twain's famous remark was applicable: "Reports of my death are greatly exaggerated!"

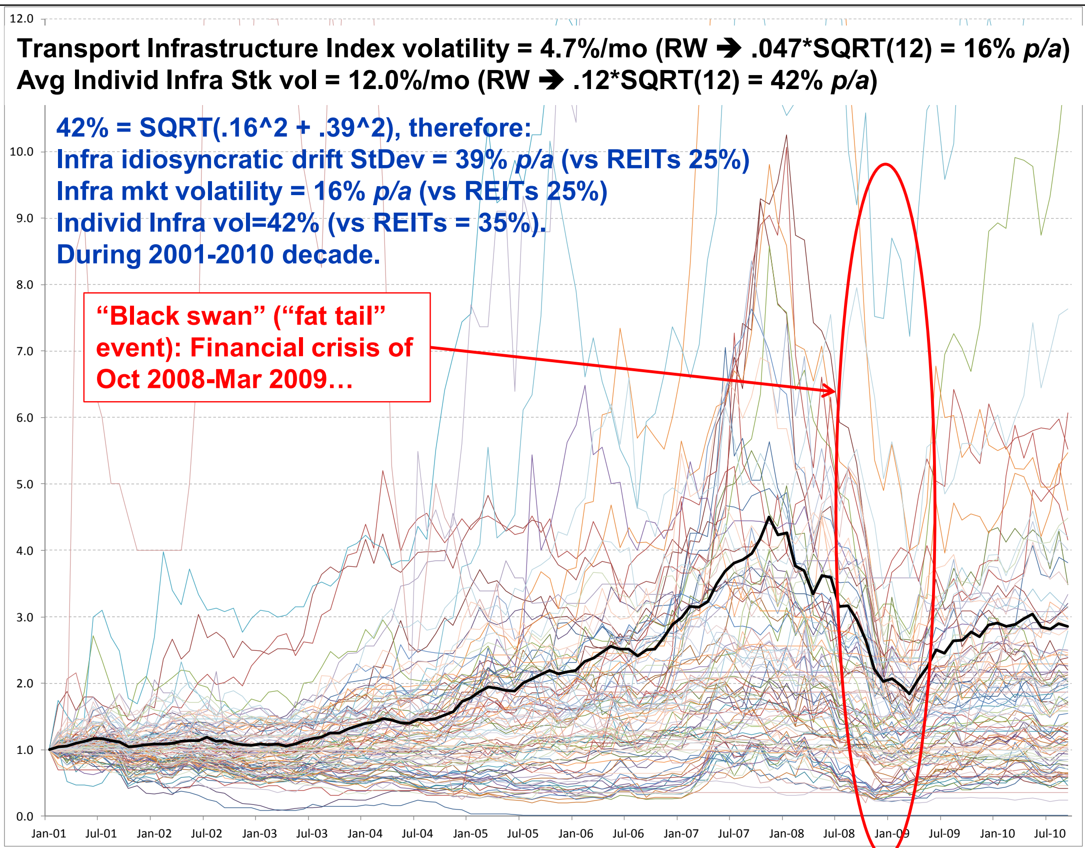
The stock market also can provide quantitative empirical evidence about the magnitude of asset value volatility among infrastructure assets. Up to now our discussion about quantifying the key asset value uncertainty relevant for major urban development projects has focused entirely on real estate. But the magnitude and often pioneering nature of such projects means that they almost always involve substantial infrastructure components, possibly including transport, water, power, and telecommunications. During most of the 20th century most infrastructure assets were publicly owned and not traded at all, making it difficult to estimate the magnitude of valuation risk in a manner comparable to that for real estate assets. However, since the latter decades of the 20th century there has developed a growing movement to privatize major infrastructure assets.

In a recent study, Rothballer and Kaserer (2011) have performed a comprehensive analysis of the volatility of an exhaustive universe of over 1400 publicly listed infrastructure firms worldwide. In general, they find that such firms display levels of individual stock risk (that is, total risk including systematic plus idiosyncratic volatility) similar to the average in the stock market. Interestingly,

[31] Comparisons between public and private markets are fraught with "apples-vs-oranges" issues. One such is in comparing "liquidity". This term tends to mean different things between the two market structures, and investors and traders tend to care about "liquidity" in different ways and for different reasons. For example, small REITs may suffer an illiquidity discount in their equity share values, even though they are by some definitions more liquid than large privately held properties that reap a premium in their valuations because they are viewed as being "more liquid" in the institutional asset market. We should also note, again, that the 2000s were a relatively volatile decade by historical standards. (Although, as we suggested at the outset of this paper, greater volatility may be characteristic of this century.)

they find that infrastructure stocks tend to display greater idiosyncratic risk than the typical stock, with less systematic or market risk (resulting in total risk that is about average). They find average individual infrastructure firm annual volatility in the order of 41% (measured in local currency). Among the infrastructure sectors examined, transport firms (airports, ports, highways, railroads, pipelines) showed 40% volatility, telecommunications firms 51% volatility, and utilities (electricity, water, gas) 33% volatility (including only 30% for water companies).

For applicability to specific project analyses the Rothballer-Kaserer figures bear the same sorts of caveats as our previous discussion of REIT volatility for real estate projects. Stock market based volatilities reflect the levered equity of firms that may hold multiple individual assets (including green-field development projects as well as stabilized income-producing assets), that employ considerable debt in the firm's capital structure, and which are viewed by stock market investors as individual entities with possibly strong active management level risk components. While we don't have private infrastructure valuation series to confirm, we would speculate based on the real estate analogy that the resulting stock market based infrastructure volatility is probably greater than that of passive individual infrastructure assets valued privately or in a direct private context (as distinct from a highly liquid public stock exchange). Nevertheless, the comparison between infrastructure stocks and REITs is at least strongly suggestive of the relative nature of infrastructure and real estate risk and uncertainty.



Source: Christoph Rothballer (TUM, 2011).

Individual Price Histories of 143 Transport Infrastructure Stocks
From 2001 Through September 2010
(Relative to Value as of December 2000 = 1.00,
Black Line is Unweighted Average Across All the Stocks)
Source: Authors
Exhibit 10

Exhibit 10 is constructed for transport infrastructure firms from the Rothballer-Kaserer data in a manner very analogous to Exhibit 9 previously described for REITs, and presented on the same scale.[32] As in the REIT Exhibit 9, the heavy black line in the infrastructure chart in Exhibit 10 represents the systematic or aggregate "sector risk" component. This is simply an index of the equal-weighted average across all of the transport firms in the chart. This transport infrastructure index volatility is lower than the corresponding U.S. REIT sector risk (16% versus 25% annualized volatility during the 2001-10 period covered). But in contrast, the individual firm volatility is larger for the transport firms than for the REITs (42% versus 35%).[33] At a minimum, the suggestion is that infrastructure components of major urban development projects probably are subject to at least approximately as much uncertainty and value volatility as are the corresponding real estate assets of such projects. Thus our previous discussion of real estate uncertainty quantification may be generally applicable to infrastructure as well at least at a broad-brush level.

This completes our "tour" of the nature and sources of real asset uncertainty and volatility, and the recently available or recently matured data sources that give us a better foundation than ever before for quantifying the uncertainty input that is vital in the flexibility valuation models discussed in earlier sections of this paper. While it is important to understand and consider all of these types and sources of real asset volatility, it is equally important to note that in actual modelling practice it will not always be necessary or desirable to separately and explicitly model each of the six sources of volatility described in this section. We often gain little additional insight by separately modelling all of these different stochastic components.

Quantitative analysis of development projects is always a mixture of "art" and "science", and both of these perspectives favour parsimony and simplicity. But we believe that the depth and sophistication of empirical knowledge about real asset price dynamics that is now possible (and getting better all the time) can substantially improve the efficacy and accuracy of the modelling and analysis of the value of flexibility in development projects.

To make the points we have presented in these first three sections of this paper more concrete, and to show an illustrative application of how they may be woven together and applied in an actual real world based example, we present a specific case in the next section.

PUTTING IT ALL TOGETHER: AN EXAMPLE BASED ON THE SONGDO IBD DEVELOPMENT PROJECT

Perhaps the best way to clarify and make more concrete the main points in the preceding sections, and to show how they may come together and reinforce each other, is to show how they may be applied to a specific project and example design question in the real world. The example we discuss in this section is one we have worked with in our classes at MIT. We like it in part because it is iconic of the type of large-scale Asian urbanization that we noted at the outset of this paper will be so dominant in the development of the world's built environment in the first half of this century.

Our example project is the New Songdo City (NSC) in Korea, and in particular its International Business District (IBD) component which is one of the most ambitious and sophisticated real estate (and infrastructure) development projects ever undertaken anywhere. But while this example is at a very mega scale, please keep in mind that the general principles and analytical approach we are suggesting in this paper can be applied at various scales. To relate this example to our discussion of

[32] The authors thank Christoph Rothballer for producing the chart in Exhibit 10.
[33] It should be kept in mind that the infrastructure stock sample examined by Rothballer and Kasserer is international, including many stocks in smaller stock markets and emerging economies that tend to be more volatile than the U.S. stock market in which all of the REITs included in Exhibit 9 are traded.

simulation modelling earlier, what we are illustrating here is a high level, relatively simple representation of the project that can be communicated to top decision makers or financial backers. More detailed engineering models would then be applied to more specific design and operational aspects. The project ***design*** element that we will focus on in this example is the staging or ***phasing*** of the overall project development program, a crucial element in the implementation of a project of such scale and complexity as the Songdo IBD.

The Songdo IBD is the central core of the New Songdo City development, which is being built on reclaimed land south of Incheon, the major port just west of Seoul. NSC is linked to the new Incheon International Airport, which is only 15 minutes away by super-highway over a new bridge. The development is a key part of Korea's national strategy to build a post-industrial economy based on headquarters and information functions. The IBD is targeted to become a major business hub of northeast Asia, and the development is designed to appeal to an international population. (NSC has been designated a "Free Economic Zone" by the Korean Government, to avoid many restrictions that apply elsewhere in the country.)

Artists Impression of Songdo International Business District at Completion
Source: Authors
Exhibit 11

The IBD master plan, designed by renowned architectural firm Kohn, Pedersen and Fox (KPF), covers 1500 acres and envisions 100m sq ft of built space including 40 m sq ft office, 35 m sq ft residential, 10 m sq ft retail, 5 m sq ft hotel, and 10 m sq ft of civic uses including world-class schools and hospitals. When completed the project is expected to house 65,000 residents and be the site of 300,000 jobs. (See Exhibit 11.) The Songdo IBD features world-class international architecture and urban design, and aims to be one of the "greenest" cities in the world, targeting 80 m sq ft of LEED certified projects, and including 600 acres of open space (including a 100-acre central park), public transit, bikeways, and greywater systems. Originally projected to have a total development cost of US$20 billion, the project is now estimated to cost over US$35 billion. The iconic 1000-foot KPF/Heerim designed North East Asia Trade Tower (NEATT) mixed-use skyscraper at the centre of the CBD was completed in 2011. By 2010 over US$3 billion funding had been secured and invested, and the success of the initial developments was providing additional cash flow to finance further development. A world-class convention centre and over 4 m sq ft of residential and retail development had been completed and successfully sold and several major office projects were nearing completion. The project could be substantially complete by as early as 2016.

For our example application of flexibility modelling, we will step back in time, to late 2002, when the project was still in its inception. At that point, the physical plan and components of the project had been pretty well established, and a key issue was the programming of the development. How should the project be staged? Should it proceed all at once as fast as possible? Or should it be broken into phases, and if so, how many phases? By breaking a large development program into explicit phases, additional flexibility is obtained. There are more opportunities to delay or abandon or simply to fine-tune exactly when and how the project proceeds. How valuable is such flexibility? How sensitive is this value to the degree of phasing (the number of phases)? These are the program design questions that our illustrative example analysis will focus on.

The first step is to build a model of the project that reflects the real option that actually exists in the ability to delay or even abandon portions of the project. In fact, only such a model can provide a rigorous and realistic evaluation of the project. Traditional DCF modelling, based on a single projected (most likely?) cash flow stream, deterministically (but artificially) fixed in time, cannot possibly realistically model the net present value (NPV) and meaningful rate of return for such a large and complex project as the Songdo IBD.

Songdo IBD Project Initial Cash Flow Projection as of 2002 (stylized):			
Year:	Costs (USD billions):	Income & Sales	Net Cash Flow:
2003	$257	$0	-$257
2004	$354	$406	$52
2005	$1,618	$841	-$777
2006	$2,637	$1,788	-$849
2007	$3,121	$2,109	-$1,012
2008	$4,153	$3,852	-$301
2009	$4,163	$3,881	-$282
2010	$2,447	$2,373	-$74
2011	$831	$1,920	$1,089
2012	$0	$7,316	$7,316
2013	$0	$6,090	$6,090
2014	$0	$4,488	$4,488
Undiscounted Sum	$19,579	$35,063	$15,484
NPV @ 20% OCC:			$1,291
IRR:			29.35%

Songdo IBD Project Initial Cash Flow Projection as of 2002 (Stylized)
Source: Authors
Exhibit 12

The table in Exhibit 12 portrays an approximate and stylized representation of the overall project cash flows as of our subject point in time.[34] This is the type of projection on which a traditional

[34] Source: Kang (2004). DISCLAIMER: Please note that while the numbers in this exhibit, and in the entire illustrative analysis to follow, are believed to be broadly similar to the actual projections used by the principals at the time, these are hypothetical numbers provided for illustrative purposes only, to demonstrate the methodological points of the current paper. The analysis herein should not be taken as an exact historical account of the Songdo IBD project in the real world, nor as an implication of either good or bad decision making on the part of any of the principals.

DCF investment analysis would be performed.[35] How might such a traditional analysis have looked at that time?

The Songdo IBD project was pioneering in several ways. Not only was it of an unprecedented scale, being built on a new site and untested location, but it was to be led by an international developer. In fact, for the first time in Korean history, a foreign developer would be allowed to purchase and own the land beneath the project. To incentivize such pioneering foreign participation and investment and jump-start the strategically important project, the Korean Government was in effect offering the developer an option to purchase land at pre-fixed prices that were designed to make the project economically appealing. In the early 2000s it had not been easy to find an international developer willing to take on the project. Considering the above, it seems likely that the project would have presented the developer with what would pencil out at a positive NPV. Indeed, we see that the projected net cash flows in Exhibit 12 provide a positive NPV of some US$1.3 billion using a 20% discount rate. Such a return was in fact a typical expected return for a speculative development project in the early 2000s, in the U.S.

However, for such a pioneering and overseas investment (from the perspective of an American developer), in a country that was at that time still perceived as an "emerging market" and not that far removed from a major financial and economic crisis (which had occurred in 1998), it seems plausible that the opportunity cost of capital (OCC) could have been realistically regarded by the developer as substantially greater than 20%. Indeed, the projected cash flows in Exhibit 12 present a going-IRR of over 29%. Thus, at a 30% OCC the project does not provide a positive NPV. Assuming a hurdle rate somewhere between 20% and 30% the project probably pencils out acceptably, and this would be a typical type of economic reasoning behind the decision to undertake such a project. But such an approach is admittedly simplistic and incomplete, and it does not allow for a rigorous exploration of differing design options, including the value of flexibility in the project, such as a consideration of the staging of the project

To do a more complete and realistic analysis and evaluation of the investment presented to the developer by the Songdo project we need to explicitly recognize the optionality that is in the project. The cash flow stream shown in Exhibit 12, while it may be a good representation of the ***expected*** cash flows (the mean of the cash flows' *ex ante* probability distribution) as of the initial decision time (end of 2002 or beginning of 2003), does not suggest the optionality that actually exists. Once the decision trigger is pulled and the project is begun, the development project is still not irreversibly committed to incur the entire cash outflows in the cost column of Exhibit 12 at the times indicated there.

Suppose it were. Then we would discount those costs to present value (as of end of 2002) at a low OCC reflecting little financial risk in that cash flow stream.[36] In 2002 this would have meant a discount rate around 5.5%. This would give the cost stream a present value of US$15.2 billion. But under such an irreversibility assumption, in order to be consistent, we should discount the benefit stream of expected cash inflows at an OCC reflecting the risk in the real estate market, the volatility in the rents and values of the assets to be built. In Korea at that time typical expected returns on real estate investments (at the property level, that is, unlevered) were in the order of 14%. Discounting the inflows at 14% produces a present value of only US$13.8 billion (even though the undiscounted

[35] The costs in Exhibit 12 include land costs at the generally pre-fixed prices being offered by the government to the developer. Thus, the NPV of the net cash flow stream in Exhibit 12 is net of land cost.
[36] Even if the cost projection is not fixed by contract, it typically would vary *ex post* primarily as a result of engineering and construction cost factors that are generally not highly correlated with financial markets, hence giving such a cash flow stream little systematic risk of the type that is priced in the capital markets. (For example, using a CAPM based model of the relevant OCC, the "beta" of the construction cost cash flow stream would be very low, calling for very little risk premium in the applicable discount rate.)

sum is over US$35 billion). Thus, to be consistent with economic theory, assuming an irreversible commitment to the project as projected in Exhibit 12, the NPV facing the developer would actually have been negative US$1.4 billion!

To construct a more complete and realistic model of the investment value of the Songdo project as of the beginning of 2003, including consideration of the flexibility value to delay or conceivably abandon later phases of the project, we first built an economic real option model of the project, of the general type described in the first section of this paper. In this case, we employed the binomial lattice type model that is common and widely taught in academia.[37]

As noted in previous sections, a crucial input to the modelling of the value of flexibility is the quantitative estimate of the magnitude of uncertainty. In the case of the economic real option model, this boils down essentially to the assumed volatility of the real assets of the type that will be built by the development project. Here is where we can use the relatively recent data and discoveries about real asset volatility discussed in the previous section. First, recall that the long-run history of major assets in the private market represented back in Exhibit 6 indicated an annual volatility in the aggregate in the private market in the U.S. of around 10%. However, we noted that during the most recent decade (which is when the most data and best quality data is available) the annual market volatility was 15%, and it may be that this decade will be more indicative of the magnitude of volatility typical in the 21st century.

Furthermore, due to the substantial positive serial correlation (inertia) in the market index, and its strong cyclical tendency over the long run, the effective volatility relevant for medium to long-run investments is probably greater than the simple annual volatility. A final consideration is that, at least as of the early 2000s, volatility in Korean asset markets probably tended to be greater than in the U.S. (at any rate, greater than the nationwide aggregate U.S. market, due to greater size and diversity within the U.S. national market[38]). For all of these reasons it would seem reasonable to take the 15% number rather than the 10% number as our indication of relevant asset market volatility for our Songdo modelling purposes.

In addition, we noted in our discussion of Exhibit 8 that typical idiosyncratic volatility has been found to be 12% pa in the U.S. data. While ideally we would like a comparable Korean data source, for present purposes we may content ourselves with the assumption that there is no obvious reason why this parameter would be different between Korea and the U.S.[39] Putting the market and idiosyncratic volatility together, we arrive at a real asset volatility estimate of approximately 20% per annum: $SQRT(0.15^2 + 0.12^2) = 0.192$. While 20% is a good point estimate, in implementing the model we also conduct sensitivity analysis by examining alternative volatility input values.

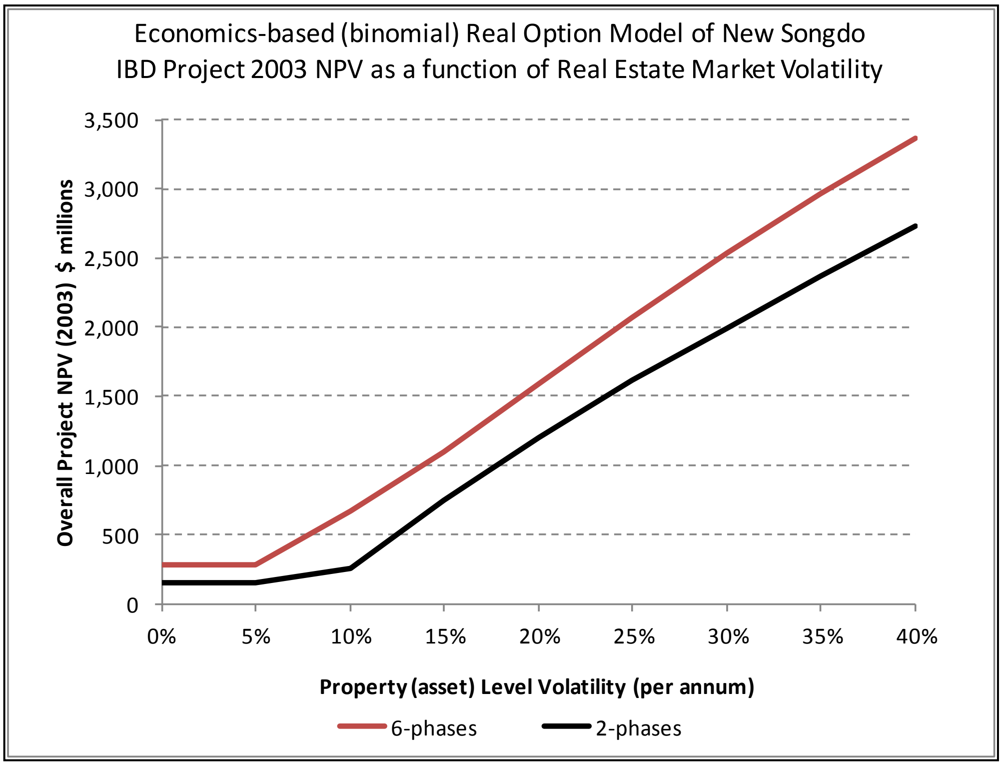
[37] This type of option valuation model is relatively intuitive and can be easily programmed in Excel®. The procedure requires specifying the volatility and the net cash yield or payout ratio of the underlying real estate assets once they would be built and operating. While the model is often specified using a mathematical device known as “risk neutral dynamics”, we employed a more intuitive (mathematically equivalent) form of the model that employs certainty-equivalence discounting. In this form the dynamics of asset valuation (the underlying real estate asset future return probability distribution) is expressed in “real” terms, reflecting actual expected price trends and probabilities for the assets to be built, enabling us to more directly use the empirical knowledge about real estate markets described in the paper, and enabling the model to show the actual expected returns for the option investments and the actual probabilities of exercise and of ex post value distributions.

[38] For example, comparisons between U.S. and British national commercial property indices reveal greater volatility in the U.K., probably at least partly reflecting the fact that the U.K. is a smaller less diverse market.

[39] This assumption is also supported by the fact that the idiosyncratic volatility estimate is rather robust in the U.S., tending to be nearly the same in a number of different markets and sub-indices, when estimated in the same manner based on residuals from repeat-sales indices.

In applying the real option model to the Songdo project we considered two design scenarios: a two-phase program and a six-phase program, to explore the nature and value of staging flexibility for the project. The result is shown in Exhibit 13 as a function of the assumed volatility of the underlying real estate assets that would be built in the project. Assuming a real asset volatility of 20%, the value of the option (i.e., the NPV of the development project net of land costs) is approximately US$1.2 billion with the minimal staging flexibility suggested by having only two phases, but US$1.6 billion with the more substantial flexibility by dividing the project into six phases, a difference worth some US$400 million or about a quarter of the more flexible project's NPV.[40]

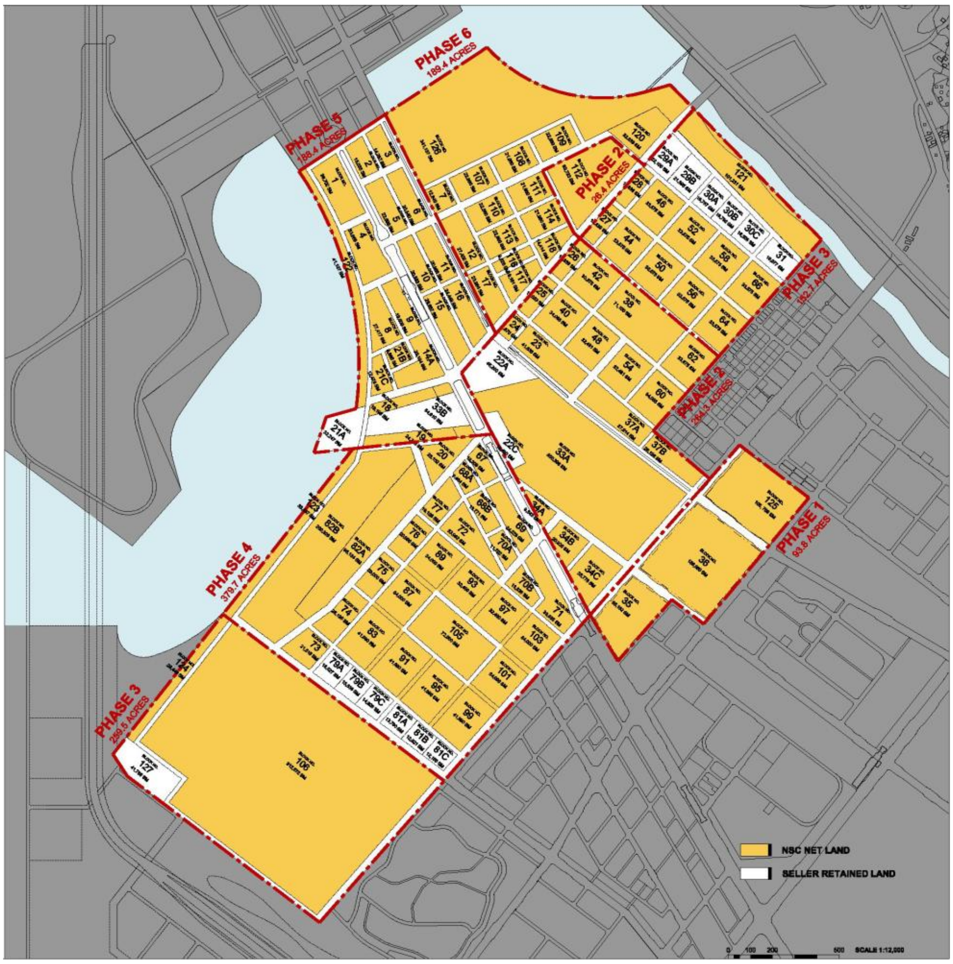
The positive NPV in 2003 was therefore roughly 6%-8% of the projected undiscounted total development costs (including the pre-specified land cost), a substantial incentive for the pioneering developer, but not outrageous by any means. The binomial model also reveals that the actual going-in expected return for the development project investment was on the order of 50% per annum for the first phase, considering the optionality (45% in the less flexible case).[41] And the model reveals that with the six-phase staging it is optimal to begin development right away in 2003 (as was in fact done), whereas with only two phases (which includes a much larger first phase) it would not be optimal to commence development right away but rather to wait initially before committing to such a large initial single phase.



Effect of Real Asset Volatility and Project Staging on NPV Based on Economic Real Option Model for Songdo International Business District Project as of End of 2002
Source: Authors
Exhibit 13

[40] The actual project was in fact divided into six phases. To model the 2-phase alternative, we started from the actual six phases and collapsed the first three into a larger first phase and the latter three into a large second (final) phase.
[41] The option model quantifies a different OCC for each phase as a function of the current value of the underlying assets and the remaining time left in the option, reflecting different amount of risk in each such "state of the world". In other words, there is no single correct OCC or discount rate for a project like Songdo, as the risk effectively changes with time and the situation in the underlying real asset market. What the economic model is telling us is that, at the very beginning, the risk was very high, but so was the expected return.

While the economic real option model is certainly of interest in its own right, as noted in the first section it is difficult for decision makers to relate to this type of model, and it has limited flexibility to realistically reflect specific aspects of the project and ways that decisions will likely realistically be made regarding the execution of the various phases of the project. Both of these shortcomings can be addressed by use of a Monte Carlo simulation model more in the classical tradition of decision analysis, the “engineering model” approach described in the previous section of this paper. The economic model can be used to help calibrate and confirm the economic validity of the engineering model, even as the latter model opens up new intuitive perspectives on the investment decision and the program design. High level screening models of the type developed in this case can be implemented in basic Excel®, using the Data Table (What-if Analysis) utility to implement the random number generation based Monte Carlo simulation.



Initial Six-Phase Staging Plan of the Songdo IBD Development Project (2002)
Source: Authors
Exhibit 14

In the present example we constructed two different versions of the project program, both based on and consistent with the original projected cash flows shown previously in Exhibit 12. In one version we grouped the cash flows into only two phases. The other version reflected the six phases that the actual real world project eventually adopted, as indicated in Exhibit 14. Each of those six phases involved the complete development of a specified non-overlapping set of land parcels (at a pre-specified price from the Korean Government), involving a mixed-use component of the overall project master plan. To construct the two-phase version, we combined the first three, and the last three, of those six phases into two much larger phases (the same as in the previously-described binomial model). In all cases the phases add up to the same projected cash flows and time line consistent with the original base case "deterministic" (most likely) cash flow projection of Exhibit 12.

For simplicity we keep the base case cost projections, but we model the uncertainty and volatility in the real asset market by allowing the outcome real estate sales and income values to vary randomly, a different random *ex post* history generated in each of 2000 runs of the Monte Carlo simulation.[42] As a benchmark, we model an "inflexible" (and therefore unrealistic) program by forcing the original base-case development timing no matter what random future history occurs. To model the effect of flexibility we build optionality and decision rules into the Excel workbook, applicable in either staging program (2-phase or 6-phase). The major decision rule extrapolates the current randomly generated real estate market outcome forward as of each potential phase start date and on that basis computes a projected IRR for that phase. If the projected IRR is above an input 30% hurdle, then the phase is triggered. (The level of the hurdle is an input parameter that can be changed.) Otherwise the phase is delayed, and the projection and hurdle criterion is checked again the next year (after a new random real estate market result is generated). If the hurdle is not exceeded after three years (four in the case of the 2-phase program), then that next phase and all subsequent phases are abandoned.

Although simplified, this structure is a reasonable model of how development project decision-making actually occurs in the real world, and reflects the essence of the process for the purposes of the screening model. An advantage of the simulation approach over the economic model is that one can model uncertainty in more nuanced and multi-faceted ways, and we can model the staging decision making process more realistically. In the present example we contented ourselves with including only the two major forms of real estate market uncertainty: asset value volatility and pricing noise. As suggested in the previous section, these two sources of volatility are often sufficient to capture most of the essential effects of uncertainty.[43] Based on the findings and analysis reported in the previous section and discussed earlier, we set these parameters at 20% for the asset volatility and 10% for the noise.[44]

In addition, unlike the economic option model, the engineering model of project NPV requires that we input a discount rate, presumably an opportunity cost of capital (OCC) relevant for the development project. From a rigorous economic perspective this is a simplification, as discussed above, as there is not a single OCC for a complex project with optionality. But it well reflects the way investors think and make decisions in the real world. In the context of the simulation model, this discount rate is a device to compute what is effectively an *ex post* NPV, that is, what the NPV (as of project inception in 2003) would be given a particular randomly generated future history. The discount rate so employed, along with other inputs in the model (such as the hurdle criterion for phase commencement), can be calibrated so that the simulation model results are broadly consistent with the more rigorous but more abstract economic real option model result noted in Exhibit 13. Based on such calibration and our discussion at the outset of this section regarding the political and economic context of the Songdo project in 2003, we set the discount rate at 25%.

[42] We experimented with a more complex model that allowed for randomness in development costs, but the results were essentially the same as reported here.
[43] The model we used is also designed to easily represent one-time "black swan" type events, though we have not explored this issue in the present research.
[44] In the paper we reported evidence of somewhat greater noise, around 15% standard deviation. However, this reflects residuals from a "mass appraisal" type model (or repeat-sales index), and we noted that it is likely that valuation estimation noise can be reduced somewhat below that when applied to specific assets, and we noted that there is some evidence for the 10% figure.

Let us now summarize the screening model simulation. Two thousand random "future histories" (unfolding commencing in 2003) of the underlying real asset market are generated based on the 20% volatility and 10% noise uncertainty parameters. In each of these "histories" an *ex post* NPV for the project is computed using the 25% discount rate. This NPV is computed in each history for each of three different development program flexibility structures:

1. no flexibility (the project is built out according to the Exhibit 12 base case projection no matter how the history unfolds);
2. flexibility with possibility for delay or abandonment at the beginning of each of two phases (based on the 30% expected return hurdle); and
3. flexibility as above only with the six phases that the actual real world project was broken into.

Simulation Results NPV Statistics (USD millions):			
Across 2000 simulated histories	Inflexible	Flexible 2 phases	Flexible 6 phases
Mean (ENPV)	$554	$1,040	$1,224
Maximum	$24,759	$24,759	$17,816
Minimum	-$6,372	-$3,058	-$2,400
Std.Dev	$3,408	$2,739	$2,743
Pct abandoned	NA	9%	0%

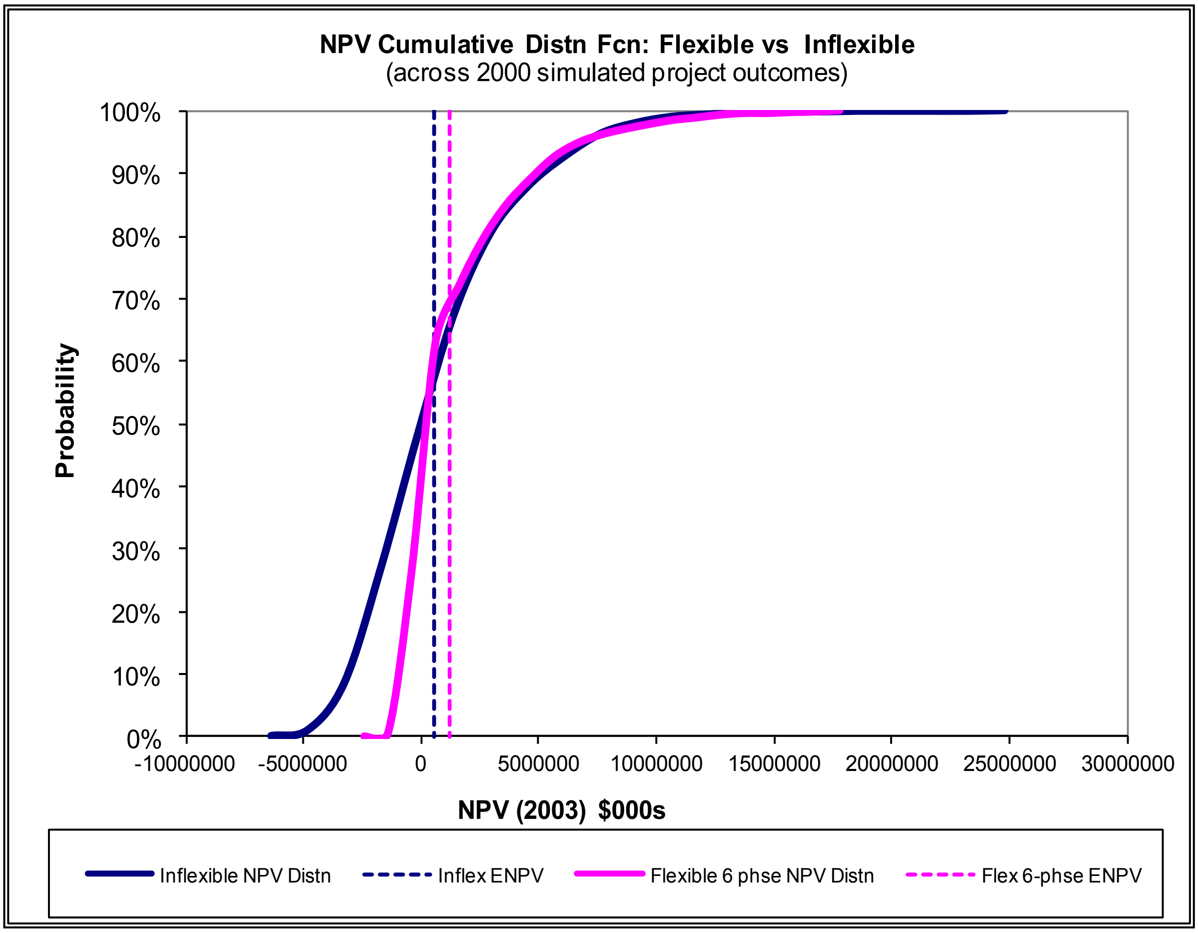
Engineering Model Results Summary for NSC
Source: Authors
Exhibit 15

The results of the simulation analysis are indicated in Exhibits 15 and 16. Across the 2000 histories, the (unweighted) average *ex post* NPV (@ the 25% discount rate) is over US$1.2 billion for the (actual real world) six-phase flexible case, almost US$200 million less at slightly over US$1.0 billion for the 2-phase flexible case, and less than US$600 million for the inflexible case. Using these average *ex post* NPV results, the engineering model provides valuations as of the end of 2002 that are broadly similar to those of the economic real option model (whose valuations are *ex ante*), but the engineering model's average NPVs are a bit lower than the economic model's. To us, this makes sense (and indeed this is part of the calibration of the engineering model[45]), as the economic model presumes optimal behaviour in terms of development timing and/or abandonment. It is more likely that the real world is not quite so optimal, and is better reflected by the decision process represented in the simulation model (to wit: commitment to each phase based on a projected IRR exceeding a typical hurdle rate, based on the then-prevailing real estate market).

As noted in the previous section, an important focus of the engineering model is that it presents not just a single *ex ante* valuation result, but an entire distribution of *ex post* results across the 2000 random histories. Thus, we can compare design alternatives not just on the basis of their average or statistical expectation of their NPVs, but also by the other characteristics of the Monte Carlo generated probability distribution of the outcomes, such as the range or standard deviation, as well as the extremes or tails of the distribution including possible skewness.

[45] As noted, the economics-based real option model is used to confirm and calibrate the engineering-oriented simulation model. (See also Masunaga 2007.)

This is shown graphically in Exhibit 16, which reveals how the flexibility to delay or abandon phases of the project cuts off the left-hand tail of the possible outcomes, leaving the project outcome distribution with more of a positive skew as upside outcomes can be taken advantage of while downside outcomes are avoided or mitigated through the technique of delaying or abandoning development phases. In our experience, decision makers and investment backers of development projects can relate to this kind of model better than they can to the more abstract economic real option models, though as suggested, it is wise for analysts to consider both and to use insights from the economic model to help hone the simulation model.



Engineering Model Results: Expectation and Cumulative Probability of *Ex Post* Songdo IBD Net Present Value Discounted at 25% PA
Source: Authors
Exhibit 16

CONCLUSIONS

The preceding sections have taken you on quite a tour. We have reviewed and attempted to synthesize three major strands of academic literature and professional practice related to the design and evaluation of investment and development in real assets, relevant for both real estate and infrastructure projects. We have presented a concrete example of how this knowledge and toolkit can be applied to analyse a major urban development project, the famous Songdo IBD project in Korea. But now we should emphasize that the intellectual enterprise that is reported on in this paper is still very much a work in process. Indeed, we feel that the profession is only at the outset of the development of the necessary data, knowledge, and tools to really improve the design and implementation of the great urban development that is ongoing apace (without waiting for us, that's for certain!), in Asia, the Middle East, and elsewhere.

There is certainly a need for more practice and honing of the art of applying the new data and tools described herein to real world projects, such as our Songdo example, but extending much further and wider into different types and scales of design questions and development problems. As described in the previous section, there is a rich and varied experience with simulation modelling of flexibility in industrial and natural resource extraction industry examples. This suggests scope for extending and furthering this "art" in the real estate and infrastructure development field.

Editor's Note: Professors Geltner and de Neufville's substantial paper has been divided into two parts. The first part (Vol 18, No 3, pps 231-249) includes the introduction and consideration of economic real option models, engineering models and Monte Carlo simulation. The second part (Vol 18, No 3, pps 251-276) includes a consideration of quantifying uncertainty or volatility through real asset pricing data and indexing, an application to Songdo New City and conclusions. For ease of reference, the abstract and references have been included in both parts.

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