VALUING REAL ESTATE AS DIVIDEND PAYING STOCK WITH A PUT-OPTION

DOUW BOSHOFF
University of Pretoria

ABSTRACT

Option pricing theory is familiar in the property market with studies that consider the methodology for the pricing of mortgages, development land, idle time, property stocks, lease options and other optionality with regards to design or use. It has not, to date, been generally considered as an alternative pricing method for income producing property as a financial investment.

This study conceptualises investment property as a dividend paying stock with a put-option at reversion. It provides the possibility to identify the risks in property investment with regards to different types of tenants and lease terms, thereby enabling the distinction between the contribution of the lease and the vacant possession to the total value of the property with a view to more accurate pricing. This potentially enables hedging within a single property against certain lease attributes or market activities.

The paper is structured in three parts. The first part contains the literature survey, which considers the different aspects of existing property valuation techniques, option pricing techniques and literature that specifically deals with option pricing within the property context. The second part deals with a theoretical explanation of the use of option pricing to value real estate, with an analysis of the required information in order to apply such techniques. The last part is an empirical testing, which is based on a portfolio of properties with varying weighted average lease expiry periods.

Keywords: property options, portfolio management, property valuation, risk pricing, optionality

INTRODUCTION

Traditional property valuation techniques have evolved over many years and, as with many other theories, been praised for superior capabilities and criticised for shortcomings. With the increasing popularity of real estate as an investment class, methods of valuation are also being reconsidered and adapted to include financial and investment models not traditionally considered solely for real estate, which could keep abreast of the increasing level of complication associated with the evolving property investment market. Option pricing as a derivative valuation technique has started to be tested on various real options in property decision making and is also now being applied to the valuation of certain aspects of property.

Recent research on real estate options has focused on mortgages, development rights and lease contracts (Oppenheimer 2006) while option pricing theory has also been applied to investment options (Patel, Paxson and Sing 2005; Alexander and Chen 2012). This paper investigates the use of option pricing for the valuation of income producing property, with specific focus on the quantification of risk that is associated with a particular property investment due to the varying lease periods that exist. The paper is structured in three parts: the first part contains the literature survey, which considers the different aspects of existing property valuation techniques, option pricing techniques, and literature that specifically deals with option pricing within the property context; the second part deals with a theoretical explanation of the use of option pricing to value real estate, with an analysis of the required information in order to apply such a technique; and the third part is an empirical testing, which is based on a portfolio of properties with varying weighted average lease expiry periods, followed by the conclusion and recommendations for further research at the end of the paper.

The paper is considered to be relevant due to the criticism of existing valuation techniques and it is contended that the theory proposed could be an alternative method that addresses some of the uncertainties in the valuation of investment property. Although many previous researchers addressed option pricing techniques in real estate, this paper is considered to be distinguishable in introducing a risk evaluation of lease periods in property using option pricing theory. No other previous research that was reviewed appeared to address this specific aspect of property valuation.

LITERATURE REVIEW

Property Valuation Techniques

Liapis et al (2011) present an integrated procedure for the evaluation of real estate investments. They present the main methods for evaluation of real estate as the price to rent ratio, net present value and internal rate of return and also state that the area in which most investors use empirical data is in the required return of the investment and in the relation with the risk of the project.

In this context, Farragher and Kleinman (1996) state that the complex real estate investment decision-making process includes the following steps:

1. setting strategy
2. establishing return/risk objectives
3. forecasting expected costs returns
4. assessing investment risk
5. making a risk-adjusted evaluation of the forecast costs and returns
6. implementing accepted proposals
7. post-auditing the performance of operating investment

Artemenkov et al (2008) document the differences between professional valuation and investment financial valuation. The former include illiquid assets such as real estate with the latter applicable to assets that are typically traded in public markets. Artemenkov et al (2008) quote various authors of research on financial economics and valuation theory and highlight the differences between various value concepts, such as price, value, individual worth and market worth (also refer IVSC 2011). The differences highlighted are between the valuation of the different types of assets, which is indicated to be largely affected by economic and financial theory. Damodaran (2012), indicated certain myths about valuation, which could be manifested in bias by the valuer and stated that in order to measure the value of an asset that is inherent in the market, independent of the actions of the valuer, correspondence should be drawn between actual observed prices of identical or comparable assets which compares property specific attributes and other leading indicators such as a broad market index, comparing property variables to equity or financial investment trends.

The main difference between professional valuation and financial-investment valuation originates from the liquidity of assets, whereby a professional valuer is required to resolve the subjectivity of two interacting parties, being generally not equivalent to situations arising on the active mass markets implied in the investment-financial valuation perspective where the market price of financial assets is determined with reference to the general market environment in equilibrium (Artemenkov et al 2008).

Sirmans (1997) suggested that the traditional DCF model may be insufficient for evaluating real estate projects. Oppenheimer (2006) notes that DCF models are using risky discount rates and subjective estimates of future cash flows. Previous critiques of DCF analysis (Hayes and Abernathy 1980; Hodder and Riggs 1985) questioned the selection process of discount rates and the inability of DCF analysis to include options in the valuation of a project, while also providing an incomplete or misleading investment decision (Hodder and Riggs 1985; Sirmans 1997). Further, various authors (Hodder and Riggs 1985; Teisberg 1995) suggested variations to the DCF method in order to allow for different circumstances during the life of an investment or project.

The above suggests that investment-financial valuation techniques might provide the possibility to overcome the limitations of professional valuation techniques, but the unique characteristics of the assets that normally form the subject of professional valuation make it difficult to apply techniques used for financial-investment valuation. Valuation techniques that incorporate change over the life of an investment, such as option pricing, may potentially be used in order to address some of these shortcomings as will be discussed in the next section.

Option Pricing Theory

Option pricing theory (OPT) was first introduced in two separate papers by Black and Scholes (1973) and Merton (1973). Later the work was combined to form the Black Scholes Merton (BSM) model. Cox et al (1979) simplified the option pricing approach by introducing binomial trees. Dubil (2004) summarises the methods of valuing options in practice as:

- a closed-form solution based on the Black-Scholes hedge argument;
- binomial trees (see also Derman and Kani1994) or trinomial trees (Derman et al 1996), that approximate the closed form through a recursive numerical induction; and
- a Monte Carlo simulation that approximates the risk-neutral expected value of the payoff based on a large number of simulated paths.

A further expansion of the binomial and trinomial trees is the implied tree with node-dependent interest rates introduced by Rubenstein (1994).

Bates (2003) summarises the empirical research performed on option pricing theory and investigates the practical value of OPT, explaining the concept of derivatives and testing the use of the BSM model over a thirty year period.

Boczar (1997) explains the utilisation of options and other financial derivatives to better understand investment risks and portfolio returns, while Gertner and Rosenfield (1999) explain the advantages of option pricing and methodology. Criticism includes limitations to the pricing of investment risk in the event of financial catastrophe (see for example Lewis 2008).

The above-mentioned studies refer to option pricing for financial assets, typically traded on an exchange such as the Chicago Board of Exchange. In option pricing, the uncertainty associated with the future price of underlying assets is the most important determinant of pricing function (Bollerslev et al 1992). Seminal papers in real option pricing include Brennan and Schwartz (1985) and McDonald and Siegel (1985), (1986).

Within the ambit of real options theory, various aspects have been considered that could also have a bearing on application of the theory on real estate. Options theory is found to be especially valuable for projects that involve both a high level of uncertainty and opportunities to dispel that uncertainty as new information becomes available (Copeland and Tufano 2004). Bellalah and Pariente (2007) state that the standard NPV approach ignores the presence of information costs, but that information plays a central role in the valuation of financial assets and must be accounted for in the valuation process. The authors state that the NPV fails because it assumes the decision to invest in a project is all or nothing due to the lack of information and the cost associated with complete information, while real option valuation allows the pricing thereof based on Merton's (1987) model of capital market equilibrium with incomplete information. Schwartz and Torous (2007) observe that real option models provide economists with a better understanding of investment decision making under uncertainty.

Wu et al (2010) review research on option theory in management information systems and suggest that further research on option theory should focus on case studies to demonstrate how it fits and improves valuation models.

Option pricing in real estate

As noted above, previous research on option pricing in real estate focused mainly on mortgages, development land options and lease contracts. From these, most studies are performed on development land options (Zeng and Zhang 2011).

Alexander and Chen (2012) introduce a general decision-tree framework for valuing real options to invest or divest at the market price of an asset in a market that need not be complete. They show that OPT has the ability to offer better decision making for investment purposes with incomplete market information. Patel, Paxson and Sing (2005) consider optionality for six primary practical uses in a comprehensive RICS research paper, being property research, investment, leasing, operations, funding and strategy.

Lam (2003) determines the differences between plain-vanilla, adjustable-rate and cash-rebate mortgages by introducing trinomial trees and provides a history of options and option pricing specifically for mortgages. Kau et al (1990) link the value of mortgage-backed security to the value of the underlying security. Carron (1988) explains the valuation of mortgages where a number of different projections of economic conditions or scenarios for the life of the security are characterized by future uncertainty about economic conditions. The value of the security in each scenario is calculated and aggregated to arrive at a composite value, called expected value or option-adjusted value. Unlike fixed rate bonds, mortgage cash flows differ according to financial conditions and unlike floating rate bonds, mortgage cash flows depend not only on current financial conditions but on an entire possible sequence of conditions in the future.

Lucius (2001) investigates the transfer of general real options theory through an examination of academic results in the field of real estate development. The author concludes that highly academic-abstract results with limited practical value are generated and that a limited number of quantitative studies regarding the valuation of real estate projects with the real options method have been conducted. Lucius (2001) states that investment theory defines real estate as a triangle of space, money and time and explains the characteristics of real estate in the light of valuation methodology, that is DCF vs option pricing. The author further states that real options theory focuses on entrepreneurial flexibility rather than on the traditional characteristic features and that flexibility equals a range of options an investor can choose from, though very few empirical studies are performed to validate the theory.

Titman (1985) investigates the valuation of urban land under uncertainty by using OPT to consider the effect of insufficient information, to make informed development decisions, on land values. Turnbull and Sirmans (1990) state that vacant land itself can be viewed as an option on various development strategies given the irreversibility of real estate development, whereby the vacant land is valued using option pricing principles with short selling of buildings or land assumed to maintain the no arbitrage condition underlying OPT (also see Capozza and Sick 1994). Shilling et al (1990) further investigate development options, but focus on the time premium of development.

On the use of OPT in lease contracts, Kenyon and Tompaidis (2001) investigate the effect of idle time between leases and also find that renewal options in leases significantly increase the value of the lease while termination options slightly reduce the value of the lease. Hendershott and Ward (2000) analyse overage rent and expense stop options and, secondly, tenant expansion and cancellation options.

The literature indicates that there are various shortcomings in traditional valuation techniques that could be addressed by financial valuation techniques. For this purpose, researchers considered option pricing as an alternative to address various aspects of real estate. The background, use and functioning of OPT is extensively studied in general and although various studies were found to consider the valuation of different aspects of income producing property using option pricing techniques, the question of differing discount rates or volatility of property returns and the effect thereof on perceived value does not seem to be fully addressed by that literature reviewed.

In the next section, a different approach will be introduced for the valuation of real estate using OPT, whereby an income producing property is considered as a dividend paying stock with a put-option at reversion with the view to analysing the relativity of the contractual vs. the reversionary income of a property to the total value thereof.

Real estate as a dividend paying stock

By using traditional DCF or income capitalization techniques for valuation, possible market changes are ignored due to unknown details of future market forces. This could possibly render the valuation of little use, especially when choices between alternative investments are to be made.

In order to resolve this issue, OPT could be used to evaluate different opportunities against each other. In order to use OPT, the assumption of risk free valuation underlying OPT would be adopted, whereby the discount rate is changed to the risk free rate, which in this case is assumed to be the long term government bond rate. The risk free rate, being lower than the property discount rate, indicates the NPV at a higher level. This additional value is evaluated by OPT and is the available amount to pay for a PUT option to fix the future reversionary value. This assumption involves the following differences to a traditional DCF valuation approach:

Traditional DCF:

- Market related income, excess rent (overage) or shortfall taken into consideration for lease period.
- Estimated reversion after predetermined period, typically 10 years, or duration of longest lease.
- Discount rate is total property return, as it reflects the opportunity cost for alternative property investments and uncertainty in market behavior.
- Difference between risk free rate (as return for systematic risk) and property discount rate represents the property risk premium (which compensates the unsystematic or

Option pricing theory:

- Actual lease income.
- Duration of lease taken as valuation period.
- Reversion after the longest lease considered to be a put option.
- Discount rate is considered to be the risk free rate as the lease income is known (although in reality this might rather be the corporate bond rate if the tenant’s credit rating is known), the reversion is taken as the lowest value that would be obtained rather than an uncertain amount, due to the put option.
- Due to certainty of lease income and put option, no vacancy exists, other than the risk of default.

idiosyncratic risk).

- Allowance is made for vacancy or lost rent, factored into income and discounted cash flow.
- Option value represents the property premium, given the specific individual property attributes.

By considering property as a dividend paying stock with a put option, the interim cash flow, or lease payments that are received, could be compared to the dividends that are received on a stock. This needs to be separated from the put option to sell at a reversionary price at the end of the lease. The main difference between the lease payments and the dividends that are received on a stock is that the dividends are not certain, while the lease payments are contractual and default risk is mainly based on the credit rating of the tenant.

The present value of the lease is determined by the cash flows generated by the lease. The present value of each payment is calculated by discounting the lease payments at an appropriate discount rate. This is generally determined by Equation 1.

$$PV_1 = \sum_{t=0}^{n} CF_t/(1+i)^t \qquad \text{Equation 1}$$

where:

PV_1	=	present value of the cash flow
t	=	time period number
i	=	discount rate per period
n	=	number of periods

This Equation requires a lengthy calculation using a spreadsheet or a repetition of the present value calculation for each individual payment, especially for long leases. Equation 1 is especially useful where all payments in the cash flow are different as each payment is calculated separately. In the event that all payments in the cash flow are the same, such as a short term lease without escalations, Equation 2, as given by Smal (1998), which determines the present value of 1 per period could be used.

$$PV_2 = \frac{1 - \dfrac{1}{(1+i)^t}}{i} \qquad \text{Equation 2}$$

where:

PV_2	=	present value of 1 per period
i	=	discount rate per period
t	=	number of time periods

If the present value of 1 per period is multiplied by the cash flow in period 1, the present value of the total cash flow is determined. The present value of 1 per period is, however, only relevant if all payments are the same throughout the cash flow.

It is not always the case that all payments in a lease are the same, but that cash flows may escalate at a predetermined rate. For cash flows where every payment is escalating at an even rate and therefore no payments are the same, Equation 3, which gives the present value of an escalating annuity as per Smal (1998), is relevant.

$$PV_3 = \frac{1 - \dfrac{(1+j)^t}{(1+i)^t}}{i - j} \qquad \text{Equation 3}$$

where:

PV_3	=	present value of an escalating annuity
i	=	discount rate per period
j	=	escalation rate per period
t	=	number of time periods

Equation 3 has the limitation that it can only determine the value of an annuity if every payment is escalating, something that is seldom the case with leases, as it is often the case that payments are monthly or quarterly and escalations are annually.

Substituting Equation 2 into Equation 3, Equation 4 could be derived that calculates the present value of an escalating annuity with different payment and escalation periods. Equation 2 had to be modified in this process, to determine the future value of 1 per period, as Equation 3 assumes payments in arrears.

$$PV_4 = \frac{(1+i)^t - 1}{i} \times \frac{1 - \dfrac{(1+j)^T}{(1+i)^{tT}}}{(1+i)^t - j - 1} \qquad \text{Equation 4}$$

where:

PV_4	=	present value of an escalating annuity, with different escalation and payment periods
i	=	discount rate per payment period
j	=	escalation rate per escalation period
t	=	number of payment periods per escalation period
T	=	number of escalation periods

Equation 4 could be used to determine the value of an escalating annuity with different payment and escalating periods, as typically experienced with normal leases in property valuation, where payments are monthly but escalation is annually, by considering only the first month's payment, the escalation rate, the number of years and the number of payments per year (assuming annual escalation). This is considered to be an advancement on other methods, as such a calculation would normally have to be done in a multi-stage calculation, while Equation 4 provides a single calculation for the entire lease value.

It should be noted that the discount rate in a traditional DCF approach takes into consideration all systematic, unsystematic or idiosyncratic risk inherent in a specific property. With the OPT approach, the risk is limited to the tenant default risk, such as late payments, failure to maintain the asset, or failure to fulfil the covenants of other financial assets (refer Sing and Tang 2004; Sing 2012). The discount rate used for contractual income should, therefore, be adjusted to exclude other property specific risk such as vacancy.

Should the lease expire and the reversion is calculated after lease expiry, it reverts to property characteristics as the property's idiosyncratic risks are also then taken into consideration and tenant risk is changing to vacancy risk. The DCF approach is, however, widely criticized for its inability to accurately predict the property variables after lease expiry as it is difficult to assume what will happen to property market variables in the future, especially when long term leases are involved.

If it is accepted that the reversionary value could be predetermined by a forward sale agreement, the risk involved in the property is transferred to the purchaser in the forward agreement and only default risk in terms of the forward agreement remains.

Instead of a forward agreement, another alternative to transfer risk is an European put option with a strike price based on the current market conditions. This creates a risk-free situation for the reversionary value due to the transfer of the future risk, after expiry of the existing leases, as a result of the put option. The reversionary value is then discounted by the risk-free rate, subject to the risk of option default. This would give an increased present value to the reversionary value, but needs to be offset against the cost of the put option.

The cost of the option to sell the property (put option) could be calculated by way of the BSM-model (Black and Scholes 1973; Merton 1973), or by binomial trees (Cox et al 1979).

Binomial trees consider the future outcome of options to be one of two alternatives, while the BSM-model determines the option value by considering the normal distribution of the possible outcomes of the option. Although binomial trees are considered to be a simpler model to apply, the BSM-model is more generally applied in OPT.

According to Hull (2011) the pricing formula for the price, *p*, of a European put is that given in Equation 5:

$$p = Ke^{-rT}N(-d_2) - S_0N(-d_1) \qquad \text{Equation 5}$$

where:

T	=	time to expiry
K	=	strike price or future reversionary price after lease expiry
r	=	risk free rate per payment period
S_0	=	current estimated market value based on vacant possession
$N()$	=	normal distribution of $-d_1$ and $-d_2$

$$d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Equation 5 determines the payoff for the right to sell a stock (or the property) at a predetermined price (based on the current price, escalated at the expected market growth rate up to lease expiry), sometime in the future. The principle of the BSM-model is to determine the likely value of the stock in the future, based on the stock's volatility, time period to expiration and the current price, which then calculates the amount that a seller would be willing to take for the right to sell the stock, or in terms of this, the property.

The total value of a single tenanted property could then be determined by combining the value of the contractual income as per Equation 4, plus the reversion which is the estimated strike price, discounted by the risk-free rate, less the cost of the put option that makes the reversion risk-free. If this is combined, the value of the property (PV) is determined as per Equation 6.

$$PV = Pmt_1 \times \frac{(1+i)^t - 1}{i} \times \frac{1 - \dfrac{(1+j)^T}{(1+i)^{tT}}}{(1+i)^t - j - 1} + Ke^{-rtT}[1 - N(-d_2)] - S_0 N(-d_1)$$

Equation 6

where:

PV	=	property value
Pmt_1	=	first contractual payment
i	=	discount rate per payment period based on tenant risk profile
j	=	escalation rate per escalation period
t	=	number of payment periods per escalation period
T	=	number of escalation periods
K	=	strike price or future reversionary price after lease expiry
r	=	risk free rate per payment period
S_0	=	current estimated market value based on vacant possession
$N()$	=	normal distribution of $-d_1$ and $-d_2$

$$d_1 = \frac{ln(S_0/K) + (r + \sigma^2/2)tT}{\sigma\sqrt{tT}}$$

$$d_2 = d_1 - \sigma\sqrt{tT}$$

One of the key determinants in the option price is the volatility (σ), which is measured on the performance of the underlying asset to which the option exists, therefore the volatility of property performance needs to be considered in order to determine the value of an option to buy or sell property. Because the option exists over space that could be sold, the volatility of the price to sell the space should be measured for a single tenanted building, as the volatility of different leases would be different and therefore the combination of different leases into a whole building would result in a different σ for every different combination of leases.

The ideal would be to have a futures index whereby individual leases could be sold on a board of exchange and the actual volatility per lease, or type of lease, could be measured. Such a futures index would, however, have no risk diversification and may be expected to be extremely volatile. As an alternative, it could be possible to use corporate bond indexes and measure the volatility of the corporate bond index for a specific tenant, but this would only be possible if an accurate assessment of the tenant's credit rating could be made and also excludes any property risk that exists in the realisation of income from property leases.

In practice, there are a number of alternative methods to measure volatility in property, each with its own characteristics and inaccuracies, which may be summarised as follows:

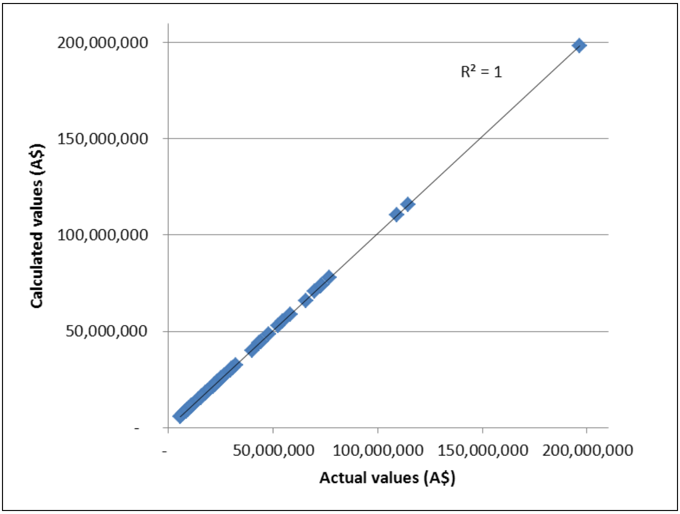
Alternative measure for volatility:	Reason for inaccuracy:
• Rent in the specific space / similar space	• Excludes capitalisation and does not reflect total price paid for income to be derived from the space in question.
• Listed property share index	• Reflects the total portfolio which might exhibit a different risk profile to the specific space in question. Also includes company variables not directly related to property itself.
• Property return index	• Reflects the total return, rather than volatility in price paid.
• Property price/valuation index	• Includes all space in any property and does not differentiate between different types of leases. Might be subject to valuation smoothing.
• A hybrid of specific property returns and a subset of property return indexes	• Needs to be developed individually and accurate information in all cases might be problematic

EMPIRICAL ANALYSIS

The proposed methodology is tested by application to the data provided by A-REITs in terms of their property portfolios owned. Sixty three properties with different attributes, owned by A-REITs and for which information is made publicly available, were considered and tested by the equations provided in the text. The data is limited to a sample of properties, rather than all properties owned, due to uniformity of information provided. The data used consists of properties for which an independent valuation has been performed on the same date, with information supplied about the valuation parameters applied. This includes the income of each property, weighted average lease expiry, discount and capitalisation rates applied, vacancy and the lettable area.

If property discount rates, as provided by the independent valuations, are applied to Equation 4, it calculates the value by way of a DCF method. The Equation is tested by applying the provided information to the Equation and the so determined value (calculated value) is then compared to the independent valuation amount as published (actual value). The calculated vs. actual values are shown in Figure 1. It can be seen that, although there are minor deviations, the calculated values are for all practical purposes identical to the actual published values, confirming the accuracy of the Equation.

The proposed alternative method is to distinguish between the contractual income, reversionary income and the cost of the put option which makes the reversionary income risk free. This would involve changing the discount rate in Equation 4 to an idiosyncratic-risk adjusted rate, which is expected to lie between the risk-free rate and the property discount rate, and change the reversionary discount rate to the risk free rate. The value is then determined by using Equation 6, which also involves the volatility of property.



Equation 4 calculated vs. actual values

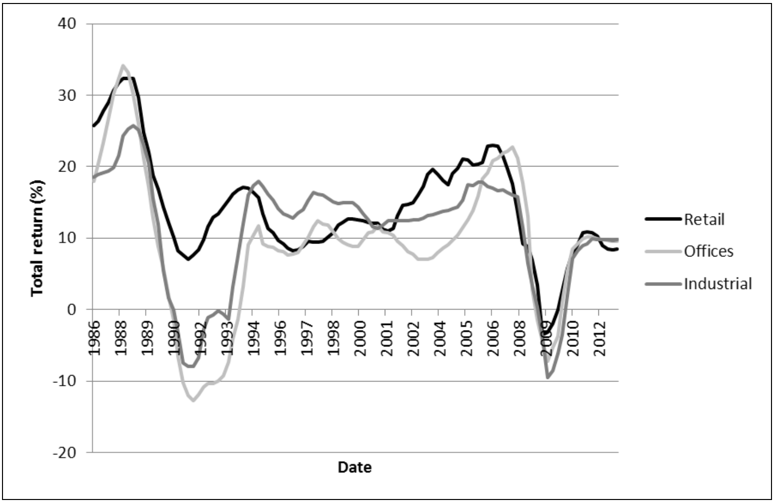
Source: Author

Figure 1

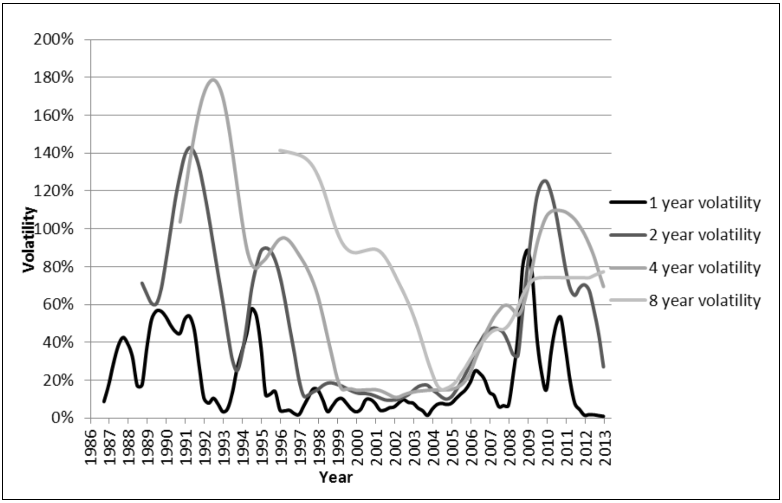
The total property returns for retail, industrial and office properties are shown in Figure 2. From this it is evident that the volatility in the returns of the different types of property differ from each other, but also differ over time. The volatility is calculated for different time periods, from 1 year to 8 year volatility, with the results for offices shown in Figure 3.

From Figure 3, the following important aspects of volatility may be observed:

- the market changes over time and the volatilities are similarly being influenced to have lower and higher levels over time;
- the longer the volatility measurement period, the more stable it becomes, due to averages being applied; and
- it appears as if the average volatility seems to be higher the longer the volatility measurement period, which is evident from the minimum, maximum and average levels of each. This would be in line with *a priori* expectation in this regard, as the longer the volatility interval, the more fluctuations are being taken into consideration, with the effect of a higher volatility measurement. It is however expected that this principle could be temporarily unstable, with sudden high fluctuations in the market return or lower in periods of stability.

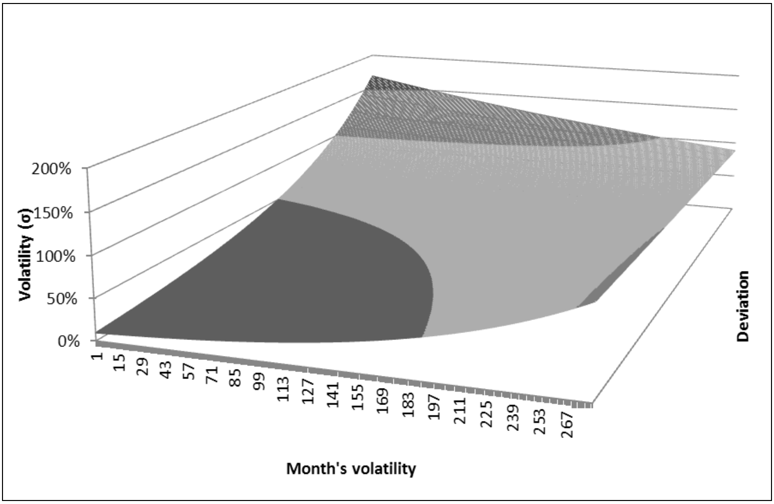


Total property return
Source: Adapted from IPD (2013)
Figure 2



Office property return volatility for different calculation periods
Source: Author
Figure 3

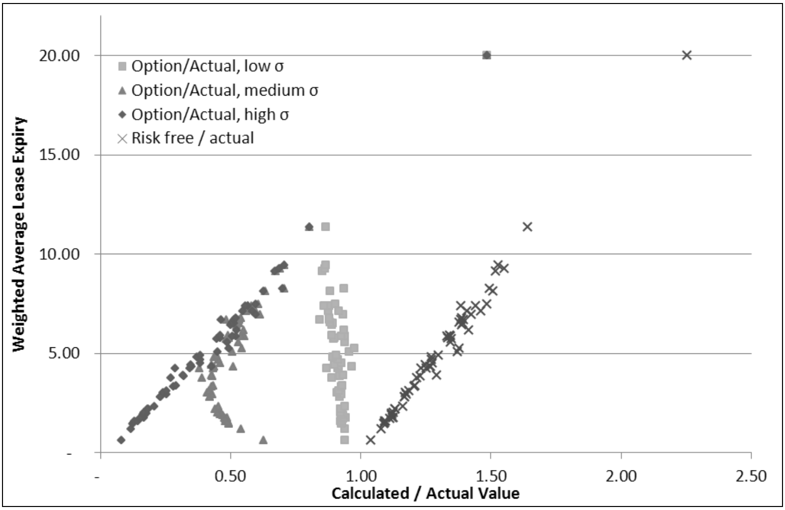
Taking the above characteristics of volatility into consideration, the volatility for office properties are estimated and shown in Figure 4, taking into consideration the lognormal distribution in terms of the average and standard deviation and the change over the time period for calculation.



Estimated office property volatility with different calculation periods
Source: Author
Figure 4

Figure 4 indicates that volatility at the lowest recorded levels is approximately 10% for short time period calculations. The highest level for short period calculations rises to approximately 180%. The difference between the lowest and highest levels becomes less over time, resulting in a small difference for calculation periods in excess of 20 years.

With the volatility known, it is possible to calculate the option price to sell the property at lease expiry for the calculated reversionary value. Figure 5 shows the total value of the properties as calculated, using Equation 6, as a factor of the independent valuation of each. There are four different results depicted, being the calculation of the value using the risk free rate as the discount rate but with no option and three results with lowest, medium and highest observed volatilities. The volatility for each data point is calculated for the same period as the WALE period. This is due to the expectation that volatility in the future over the period until expiry would be closest measured if a similar period volatility calculation of the past is used, although deviations from this rule may occur.



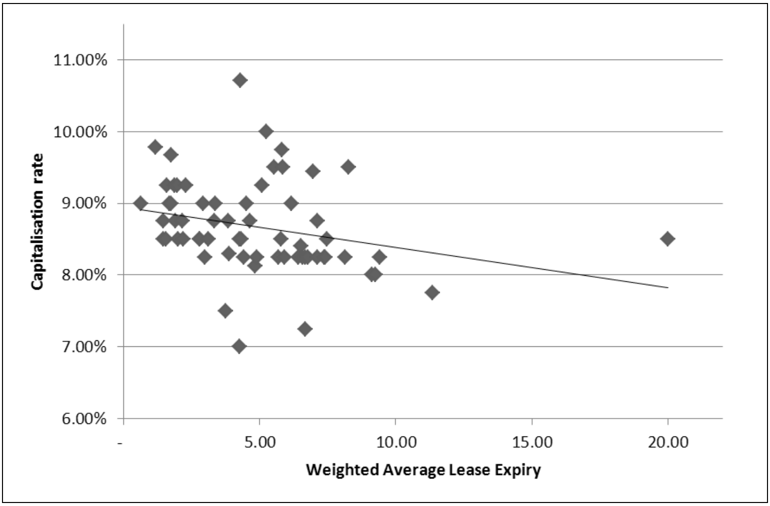
Calculated/actual at different levels of volatility vs risk-free discounted values
Source: Author
Figure 5

The results as shown in Figure 5 have a number of observations worth noting:

- for the calculations using lowest level of observed volatilities, the calculated values are very similar, albeit a bit lower, than the actual values, hence a ratio of close to 1 for all data points. From Figure 4 it is observed that volatility at this level increases for longer period calculations;
- the data set using medium level observed volatilities indicates the properties with medium term (3 to 5 years) WALE periods to be most affected by the option to fix the reversionary value. The short term (below 3 years) is affected, but it is evident that the longer the WALE period, the higher the impact, even though it is observed in Figure 4 that volatility for different calculation periods and at this level of deviation remains fairly stable. With WALE periods longer than 5 years the impact declines, up to a point where the values are almost similar to the independent calculated values for WALE periods in excess of 10 years (note the single observation of a property with a WALE of 20 years which seems to be an outlier based on the OPT calculation, refer concluding remarks in this regard); and
- with the highest levels of volatility, shorter period properties are severely affected, to the extent that the properties with the shortest WALE periods calculate to a value close to zero. The effect, however, reduces, with a lesser impact on the longer WALE periods. This would also be due to the shape of the volatility curve shown in Figure 4, where the volatility declines over time at the highest deviation levels.

The above three observations could also be considered with regard to the observations in Figures 1 to 3, where it is indicated that the short term option price (1 year) has a fairly straight line upward sloping curve for higher levels of volatility, while the slope for the medium term option price declines slightly for higher volatilities and for long term option prices, the diminishing slope starts at a lower level and evens out at lower levels of volatility than for medium and short term option prices, which confirms the findings illustrated by Figure 5.

From Figure 5 it is evident that with shorter WALE periods the calculated values are slightly less affected than properties with medium term WALE. As the WALE increases, the option values calculated for different volatilities move closer to each other up to the point where it is very similar with WALE at approximately 12 years, indicating a lesser influence of the option itself on the total value. This suggests that the longer the WALE, the lesser the cost of an option to make the investment risk free and therefore the lower the inherent property risk, as depicted by the closer results of the three data sets shown in Figure 5 for longer WALE periods. Although this could not be confirmed conclusively, Figure 6 might also suggest a similar result where the independently determined capitalisation rates of the properties are indicated to have a slight downward trend for longer WALE periods.



Capitalisation rates at different WALE periods
Source: Author
Figure 6

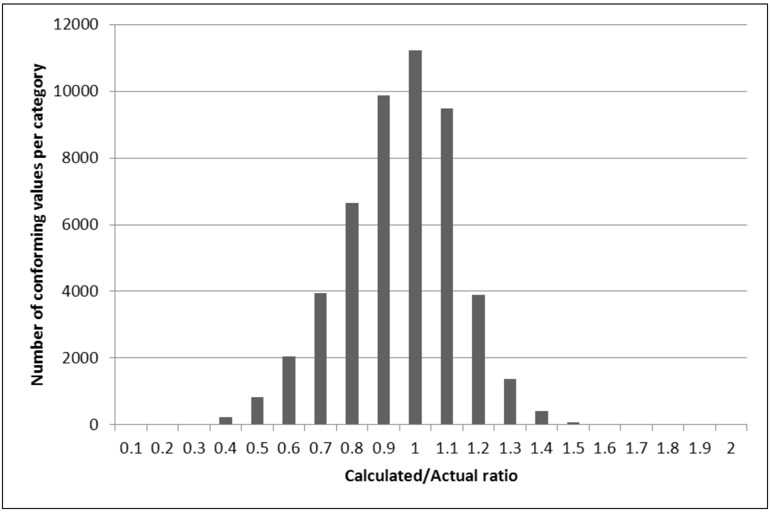
At this stage it is, however, only shown for illustrative purposes and not to make any inferences. It is also worth mentioning that very few observations were found with WALE periods in excess of 10 years and it is the norm not to exceed this. It does not, however, preclude such longer leases to be signed, but the lack of observations of such leases prevents analysis thereof in this context.

The results shown in Figure 5 assumed the discount rate for both the contractual income and the reversionary value to be equal to the risk-free rate. In reality this is expected not to be true, although there is *a priori* expectation for the rates to be lower than the property discount rate by inclusion of the put option, but not as low as the risk-free rate due to some idiosyncratic risk that remains. The importance of this is the ability to partition the risk into different risk factors. This could be done by applying a Monte Carlo simulation to the different variables in order to investigate the influence of each part of the risk on the total (also refer Sing 2012; Geltner and De Neufville 2012).

To test this, a Monte Carlo simulation was performed by assuming the following:

- volatility is randomly chosen between the -2 and -1 standard deviations for different calculation periods of the volatility shown in Figure 4;
- with the lowest volatility still providing calculated values that are lower than actual values, a random factor between 0 and 1 of observed volatility is applied in order to test the level of volatility at which the calculated values are within an acceptable range (volatility factor);
- the discount rate for the contractual income is randomly chosen between the risk-free rate and the independent discount rate, whereby a factor between 0 and 1 is applied to the risk premium (the contractual risk premium factor); and
- the discount rate for the reversionary value is randomly chosen between the risk-free rate and the independent discount rate, whereby a factor between 0 and 1 is applied to the risk premium (the reversionary risk premium factor).

The essence of the Monte Carlo simulation is that it is possible to test the outcome of different combinations of the reversionary risk premium, the contractual risk premium and volatility. This was performed for approximately 50,000 iterations (approximately 800 iterations per property). The frequency distribution with a 0.1 interval for calculated/actual values is provided in Figure 7.

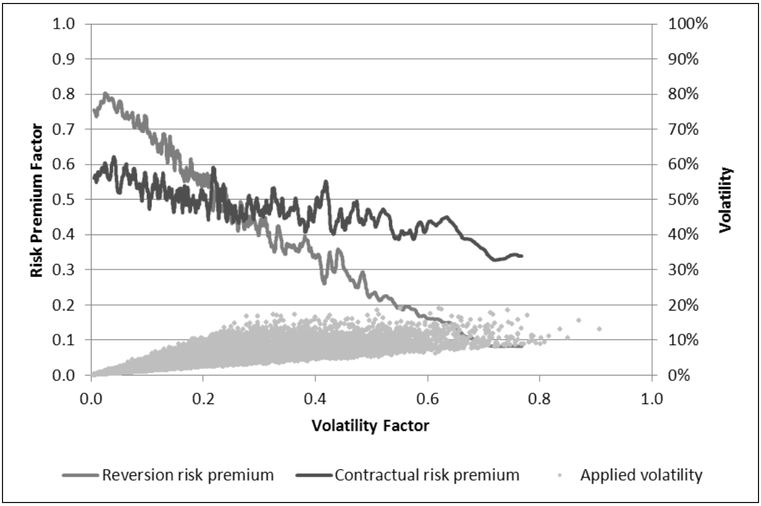


Monte Carlo simulation for different levels of volatility and discount rates
Source: Author
Figure 7

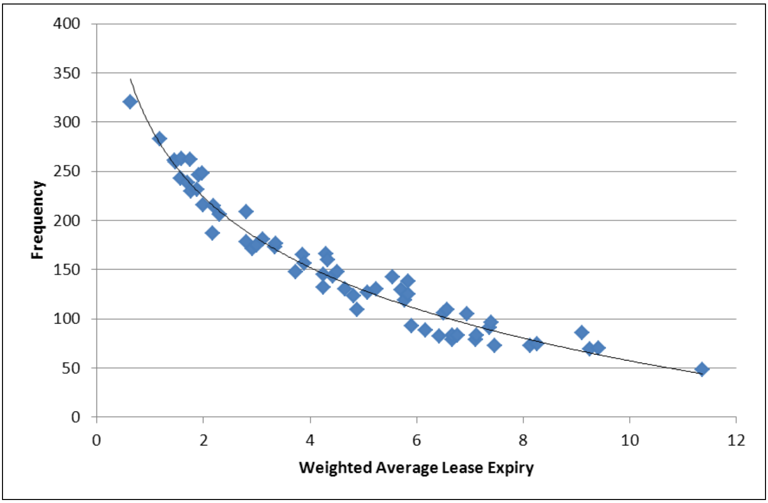
Approximately 20% of the calculated values were within a range of 0.95 to 1.05 of the actual values, which are referred to as conforming properties. The results of the risk premium factors plotted against the volatility factors are shown in Figures 9 and 10. It is evident that very little inference could be made due to the number of iterations, although the density distribution suggests a downward slope of both contractual and reversionary risk premium factors if plotted against the volatility factor. In order to better explain this, a 100 iteration average of the reversionary and contractual risk premium factors, calculated for incremental increases in the volatility factor, is shown in Figure 8 and also shown with the applied volatility.

The findings of the Monte Carlo simulation, with reference to Figures 9 and 10, may be summarised as follows:

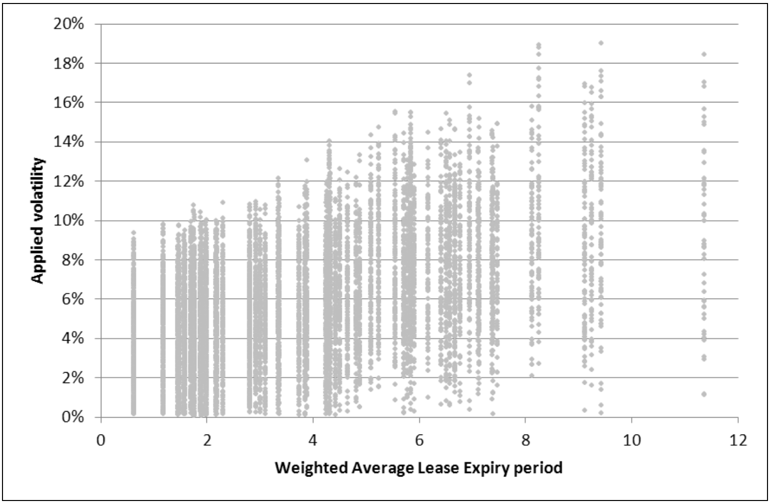
- the property with the WALE period of 20 years distorted some of the results, with an option value of zero in some cases. Thus it was excluded from the Figures and would need further investigation (refer conclusions and limitations);
- the properties that were included in the empirical analysis indicated that the market perceives volatility to be below 20%, with evidence of volatility to be close to 10%. When volatility exceeds this, the value determined by OPT becomes non-conforming;
- it is possible with the Monte Carlo simulation to observe the higher emphasis on the volatility with a reduction in the risk premiums for both the contractual income and the reversionary income, indicating the effect of the option on the discount rates used for the contractual and reversionary income;
- according to Figure 8, the contractual risk premium factor approaches 0.3 and the reversionary risk premium factor reduces below 0.1 of the property risk premium, both for a volatility factor that approaches 1. It is evident that the reversionary risk premium factor is more sensitive for changes in the volatility factor, confirming the applicability of OPT to calculate the risk involved in the reversionary value, but does not have much of an influence on the contractual income, which is also not calculated thereby. The reversionary risk premium factor appears to stabilize above 0.05 of the property risk premium, while it could also be observed that the contractual risk premium is in the order of 0.5, indicating the lower risk in the contractual income than the risk in the reversionary value; and
- the frequency of conforming properties has a negative lognormal distribution to WALE periods, while the applied volatility increases with longer WALE periods. This confirms the findings shown in Figure 3 that volatility has a diminishing effect in longer WALE periods. The higher frequency of conforming observations at lower WALE periods is due to the lower effect of the DCF on the values for shorter discounting periods, with the result that more properties are conforming, even at zero volatility. This also explains the effect in Figure 10 that properties with a WALE of less than 4 years appear to signal negative volatility for the graph to have an even distribution.



Average risk premium factors and applied volatility vs. volatility factor
Source: Author
Figure 8



Frequency distribution of conforming properties at different WALE periods
Source: Author
Figure 9



Applied volatility at different WALE periods
Source: Author
Figure 10

SUMMARY

The paper introduces a different pricing formula for valuing income producing property. The life of the property investment is separated into the contractual income, which is more certain than the other part, the reversionary income. The latter is again separated between a risk-free portion and a risky portion. This enables the possibility of evaluating the inherent risk in any property by considering the specific lease terms with regards to duration and the level of property specific risk that exists in individual investments separately, with accurate risk partitioning.

From the above it is evident that WALE periods have a significant effect on the inherent risk in a property, with properties with longer leases indicated to be substantially higher valued at the risk free rate. There is, however, only a limited change in the independent capitalisation rates for properties with longer WALE periods, which might indicate that the effect of longer leases is not fully quantified by valuers when performing valuations, but this is not conclusively tested and should be done by comparing OPT valuation to actual sales of properties with different lease durations and also consider the credit ratings of individual tenants.

The applied volatility of conforming properties was identified to be lower than the observed market volatility of property returns. This indicates that the sample used is considered less volatile than volatility of returns experienced in the market as a whole. The study was, however, performed on independently valued properties, not actual sales, and might therefore be subject to valuation smoothing. It may be contended that, even in highly volatile periods, property will trade at lower volatility levels than that of the property returns due to the expectation that returns will even out over the longer period. The study should, however, further explore the effect of actual sales on the pricing method, but was considered necessary in this form in order to test the hypothesis on uniform data.

The Monte Carlo analysis identified that the risk premium for contractual risk nears 30% of the total property risk premium as the volatility nears 100% of observed market volatility. This suggests that at least 30% of the property risk premium could be considered part of idiosyncratic risk inherent to the contractual income. This should be further explored for different types of tenants, with different credit ratings, as well as different types of property, uses and locations.

The risk premium for the reversionary income nears 5% of the property risk premium for volatility that nears 100% of the market volatility. This confirms the successful application of the OPT to partition the observed risk in the property and that the put option value can successfully explain the inherent risk of the reversionary income. This implies that, based on the sample investigated, the risk associated with the reversionary income could be as low as 5% of the property risk premium and is therefore systematic in nature, with the possibility to accurately price the risk and the option to diversify it. The feasibility of a lease futures exchange could be investigated, including the regulatory requirements of such an exchange.

With the importance of volatility as part of the option pricing equation, it is imperative that volatility be measured accurately. The contribution of this paper is in its ability to identify the risk that such volatility poses to the investor, specifically with regards to the reversionary income, while the risk characteristics of the contractual lease income should be measured separately in order to obtain an accurate perspective of the total risk involved.

CONCLUSION

The paper investigates the use of OPT to value income producing property and partition the risk involved in such properties between contractual income risk and the reversionary value risk. This makes it possible to accurately price the reversionary income risk and the option to diversify it, suggesting such risk to be as low as 5% of the property risk premium and indicating the reversionary income to be systematic in nature. The paper points out the importance of lease durations on the risk involved in property investment and indicates OPT to be a viable alternative to calculate the influence of lease duration on the value of a property. The paper also measures the effect of volatility on property values, with an indication of the level of volatility that is comparative in explaining property values that are independently valued.

The value of the paper is in that it introduces a method to better understand risk in any property investment and introduces a method to partition risk into identified categories.

The paper is limited in the sense that it uses independent valuations as reference and should also consider actual sales. It was further limited by the very small sample of properties with WALE periods above 10 years, causing a gap in the understanding of the model for WALE periods above 10 years. The single sample with a WALE period of 20 years seems to be virtually unaffected by the option, with the pricing calculation indicating the put option for such a property to be zero. It could be investigated how the option nears zero by taking a larger sample of properties with WALE periods between 10 and 20 years.

REFERENCES

Alexander, C and Chen, X 2012, *A general approach to real option valuation with applications to real estate investments*, ICMA Centre Discussion Paper, No. DP 2012-04.

Artemenkov, A, Mikerin, G and Artemenkov, I 2008, 'Professional valuation and investment-financial valuation: distinctions in valuations for private and public markets', *The Appraisal Journal*, Vol. 76, No. 4, pp. 355-370.

Bates, DS 2003, ‘Empirical option pricing: a retrospection’, *Journal of Econometrics,* Vol. 116, No. 1, pp. 387-404.

Bellalah, M and Pariente, G 2007, 'Real options with information costs: a synthesis', *International Journal of Business,* Vol. 12, No. 2, pp. 155-179.
Black, F and Scholes, M 1973, 'The pricing of options and corporate liabilities', *Journal of Political Economy,* Vol. 81, No. 3, pp. 637-654.
Boczar, TJ 1997, 'An introduction to options and other financial derivative strategies', *Trusts and Estates,* Vol. 136, No. 3, pp. 43-54.
Bollerslev, T, Chou, RY and Kroner, KF 1992, 'ARCH modelling in finance: a review of the theory and empirical evidence', *Journal of Econometrics,* Vol. 52, No. 1-2, pp. 5-59.
Brennan, MJ and Schwartz, ES 1985, 'Evaluating natural resource investments', *Journal of Business*, Vol. 58, pp. 135-157.
Capozza, D and Sick, G 1994 'The risk structure of land markets', *Journal of Urban Economics*, Vol. 35, pp. 297-319.
Carron, A 1988, 'The option valuation approach to mortgage pricing', *Journal of Real Estate Finance and Economics*, Vol. 1, No. 2, pp. 131-149.
Copeland, T and Tufano, P 2004, 'A real-world way to manage real options', *Harvard Business Review*, Vol. 82, No. 3, pp. 90-99.
Cox, J, Ross, S and Rubinstein, M 1979, 'Option pricing: a simplified approach', *Journal of Financial Economics*, Vol. 7, No. 3, pp. 229-264.
Damodaran, A 2012, *Investment valuation: tools and techniques for determining the value of any asset*, 3rd Edition, John Wiley And Sons, New Jersey.
Derman, E and Kani, I 1994, 'Riding on a smile', *Risk*, Vol. 7, No. 2, pp. 32-39.
Derman, E, Kani, I and Chriss, N 1996, *Implied trinomial trees of the volatility smile,* Goldman Sachs Quantitative Strategies Research Notes.
Dubil, R 2004, 'The risk and return of investment averaging: an option-theoretic approach', *Financial Services Review*, Vol. 13, No. 4, pp. 267-283.
Farragher, EJ and Kleinman, R 1996, 'A re-examination of real estate investment decisionmaking practices', *Journal of Real Estate Portfolio Management*, Vol. 2, No. 1, pp. 31-39.
Geltner, D and De Neufville, R 2012, ‘Uncertainty, flexibility, valuation and design: how 21st century information and knowledge can improve 21st century urban development’, *Pacific Rim Property Research Journal*, Vol. 18, No. 3, pp. 231-276.
Gertner, R and Rosenfield, A 1999, 'How real options lead to better decisions', *Financial Times*, Academic One File.
Hayes, R and Abernathy, W 1980, 'Managing our way to economic decline', *Harvard Business Review,* Vol. 58, No. 4, pp. 66-77.
Hendershott, PH and Ward, CWR 2000, 'Incorporating option-like features in the valuation of shopping centres', *Real Estate Finance*, Vol. 16, No. 4, pp. 31-36.
Hodder, J and Riggs, H 1985, 'Pitfalls in evaluating risky projects', *Harvard Business Review*, Vol. 63, No. 1, pp. 128-135.
Hull, JC 2011, *Fundamentals of futures and options markets*, 7th Edition, Prentice Hall, Boston.
International Valuation Standards Council 2011, *International valuation standards*, IVSC, London
IPD (2013), *The Property Council/IPD Australia all property index: results for the year to 31 March 2013,* Property Council/IPD Australia, Sydney.
Kau, JB, Keenan, DC, Muller, WJ and Epperson, JF 1990, 'Pricing commercial mortgages and their mortgage-backed Securities', *The Journal of Real Estate Finance and Economics*, Vol. 3, No. 4, pp. 333-356.
Kenyon, C and Tompaidis, S 2001, 'Real options in leasing: the effect of idle time', *Operations Research*, Vol. 49, No. 5, pp. 675-689.
Lam, E 2003, ‘Option pricing model for cash rebate mortgages’, *Journal of Property Investment and Finance,* Vol. 21, No. 4, pp. 307-325.
Liapis, KJ, Christofakis, MS and Papacharalampous, HG 2011, 'A new evaluation procedure in

real estate projects', *Journal of Property Investment and Finance*, Vol. 29, No. 3, pp. 280-296.

Lewis, M 2008, *Inside Wall Street's black hole*, Conde Nast Portfolio.

Lucius, DI 2001, 'Real options in real estate development', *Journal of Property Investment and Finance*, Vol. 19, No. 1, pp. 73-78.

McDonald, R and Siegel, DR 1985, 'Investment and the valuation of firms when there is an option to shut down', *International Economic Review*, Vol. 26, pp. 331-349.

Merton, RC 1973, 'Theory of rational option pricing', *Bell Journal of Economics and Management Sciences*, Vol. 4, No. 1, pp.141-183.

Merton, RC 1987, 'A simple model of capital market equilibrium with incomplete information', *Journal of Finance*, Vol. 42, pp. 483-510.

Oppenheimer, P 2006, 'A critique of using real options pricing models in valuing real estate projects and contracts', *Briefings in Real Estate Finance*, Vol. 2, No. 3, pp. 221-233.

Patel, K, Paxson, D and Sing, TF 2005, 'A review of the practical uses of real property options', *RICS Research Paper Series*, Vol. 5, No.1.

Rubinstein, M 1994, 'Implied binomial trees', *Journal of Finance*, Vol. 49, No. 3, pp. 771-818.

Schwartz, ES and Torous, WN 2007, 'Commercial office space: testing the implications of real options models with competitive interactions', *Real Estate Economics*, Vol. 35, No. 1, pp. 1-20.

Shilling, JD, Sirmans, CF, Turnbull, GK and Benjamin, JD 1990, 'A theory and empirical test of land option pricing', *Journal of Urban Economics*, Vol. 28, No. 2, pp. 178-186.

Sing, TF and Tang, WL 2004, 'Valuing leasing risks in commercial property with a discrete-time binomial tree option', *Journal of Property Investment and Finance*, Vol. 22, No. 2, pp. 173-191.

Sing, TF 2012, 'A real option approach to pricing embedded options in retail leases', *Pacific Rim Property Research Journal*, Vol. 18, No. 3, pp. 197-211.

Sirmans, CF 1997, 'Research on discounted cash flow models', *Real Estate Finance*, Vol. 13, No. 4, pp. 93-95.

Smal, C 1998, *The time value of money in practice*, Protea, Pretoria

Teisberg, EO 1995, 'Methods for evaluating capital investment decisions under uncertainty', in Trigeorgis, L (Ed), *Real options in capital investment: models, strategies and applications*, Praeger, Westport, pp. 31–46.

Titman, S 1985, 'Urban land prices under uncertainty', *American Economic Review*, Vol. 75, pp. 505–514.

Turnbull, GK and Sirmans, CF 1990, 'Vacant land options: a theoretical analysis', *Regional Science And Urban Economics*, Vol. 20, No. 2, pp. 213–222.

Wu, LC, Wu, LH and Wen, YF 2010, 'Interdisciplinary research of options theory and management information systems: review, research, issues and suggestions for future research', *Industrial Management And Data Systems*, Vol. 110, No. 3, pp. 433-452.

Zeng, S and Zhang, S 2011, 'Real options literature review', *Ibusiness*, Vol. 3, No. 1, pp. 43.

ACKNOWLEDGMENTS

The author wishes to extend his gratitude towards the Editor for very useful comments in successfully completing this paper, as well as the constructive criticism that was provided by both the Editor and the two anonymous referees.

Email contact: douw.boshoff@up.ac.za