"WHITE" SITE VALUATION: A REAL OPTION APPROACH

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ABSTRACT

The "white-site" zoning concept was introduced in Singapore to give developers more flexibility in the use of the sites they bought via the government's sale of sites program. Endowed with the switch use options, developers can respond to the market demand and supply conditions more effectively by instantly adjusting and optimizing the space among different uses without having to pay a hefty differential premium. This study proposes a two-stage sequential real option model to jointly estimate the premiums for the flexibilities in switching use and selecting the most favorable timing for a white site development. The proposed real option models are then applied to empirically estimate the premiums for five selected white sites.

Keywords: White site, switch use option, optimal development timing option, option premium.

INTRODUCTION

The concept of a "white" site was first introduced by the Urban Redevelopment Authority (URA), the planning authority of Singapore, in October 1995, to give developers more flexibility in development options on certain land parcels sold by the state through the sale of sites program. Under the URA's white site guidelines, developers are free to decide on the mix of uses and the respective quantum of floor space of each use for the site as long as the total permissible gross floor area (GFA) for the whole development is not exceeded. Unlike fixed use sites, developers do not need to pay a differential premium[1] for the change of use and the development mix use during the lease period. The flexibility given to the "white" site allows developers to strategize their development activities to the best of their interests in responding to changing market conditions. In a market with uncertain demand, this flexibility allows developers to optimize the development potential of the site.

The successful bidder of a white site has the option to develop the site for commercial, residential or hotel use, or a mix of these uses, as well as the right to choose the quantum and/or the mix of the use when initiating or launching the development. This flexibility, to switch the quantum or the mix of use from time to time, increases the development potential of the site, and hence enhances the market value of the site. For example, if a developer chooses to lock-in a proposed development for a 100% office use at the initial design and construction stage, but due to oversupply when the project is completed, he may exercise the option to convert some of the office space into retail or residential uses. This option to switch is especially valuable when the market is highly

1. Differential premium is a 100% windfall tax levied on the increase in value of a 99-year leasehold site resulting from the government's permission to allow for change of use, increase in density or extension of the leasehold period.

volatile and dynamic. If the option to switch is fairly priced into the market value, the residual land value of the "white" site will be more valuable than a normal site.

Given that the option to switch use is valuable in a "white" site, the question of how much should be the "premium" is therefore significant and of practical relevance in the valuation of a "white" site. The traditional valuation techniques such as the residual land valuation and the market comparison approach are no longer adequate or appropriate to deal with the embedded option features in the land. Real option models have been used to examine how the flexibility created by mixing use and switching use options would impact on the development option, and consequently on the land and property values. Geltner, Riddiough and Stojanovic (1996) extended the optimal development timing models of Williams (1991), Quigg (1993) and Titman (1985) by incorporating the land use choice option into the proposed development option model. In the model with land use choice of two uncorrelated uses, they found that land use choice delays development compared to single use type of development. They also showed that the value of land with multiple use zoning (two uses in the model) is 40% higher than that with single use zoning. Using the same framework, Childs, Riddiough and Trianitis (1996) examined the impact of flexibility of mixing use given by the multiple redevelopment option on under-developed or undeveloped land values. Their findings were consistent with those of Geltner, Riddiough and Stojanovic (1996), which suggested that the mix use option does effectively retard the rate of development relative to single use land in some circumstances.

While these two papers focused on the effects of land use choice on the development timing, Grenadier (1995a) further extended the model to look at how the post-development switch-use option affects property value. Based on a model with only two uses, Grenadier (1995a) showed that the switch use flexibility is highly valuable when the demand for space is highly volatile. The model quantifies the landlord's option of switching lettable space from one use to another use, subject to a fixed adjustment cost. Sing and Patel (2000) relaxed the assumptions in Grenadier's model by using two exogenous rental variables as stochastic state variables and, instead of a fixed adjustment cost, they used the incremental property value after adjustment as the boundary condition for triggering the switch use option model. The results were not dissimilar from those obtained in Grenadier (1995a), which suggests that increasing uncertainty in the relative rental level increases the value of the building with dynamic switch use flexibility.

On the development timing option, Williams (1991) and Quigg (1993) have developed two different continuous-time real option models based on the assumptions that the revenue and cost variables evolve stochastically over time. Sing (2001) also looked at the land development problems of Williams (1991) and Quigg (1993) by dealing more explicitly with the density effects of the rental revenue and development cost in a real estate project. The closed form analytical solution derived in Sing (2001) would be incorporated in the second stage of the timing option premium estimation for the "white" sites.

Based on the real option framework of Grenadier (1995a), Sing and Patel (2000) for optimal tenant mix and Sing (2001) for the optimal timing option model, this paper aims to propose a real option model to value the switch-use options embedded in "white" sites. The paper is organized into five sections. Following a brief introduction in Section 1, Section 2 gives an overview of the white site planning concept and the implications on the valuation and development processes in Singapore. Section 3 sets up the basic structure and assumptions of the model. The stochastic derivations of the switch-use option and optimal development timing option models in a two-stage process are also covered in this section. Section 4 applies the proposed real options model to empirically estimate the dynamic flexibility and the optimal timing values based on assumptions of the common and case-specific input parameters. The premiums attributed to dynamic switch use flexibility and optimal timing options are also evaluated. Section 5 concludes the findings.

WHITE SITE – FLEXIBILITY IN LAND USE PLANNING IN SINGAPORE

"White" site as land use zoning

Land use and transportation planning in Singapore is governed by a two-tier planning system consisting of a concept plan and the Master Plan. The concept plan is a long-range macro level plan, which defines the broad direction of the land use allocation and transportation networks. Under the first concept plan proposed in 1971, several major projects like the Changi International Airport, the Mass Rapid Transit (MRT) System, expressways, new towns, etc. were completed. In 1991, a revised concept plan, "Living the Next Lap", was unveiled to further transform Singapore into a tropical city of excellence in the next century. Under the revised plan, a new downtown core with four regional centers would be developed to turn Singapore into an international financial and business hub. The broad visions spelt out in the concept plan were translated into micro-level development guide plans. There are 55 development guide plans (DGPs) proposed to provide guidelines on land use and development activities in Singapore. The DGPs contain detailed specifications on the permissible development types, height of building, and density of development for every single parcel of land in Singapore. With these control parameters, the physical development activities could be guided and controlled more systematically and the urban planning process could be undertaken more efficiently. The DGPs have now been compiled as the 1998 Master Plan.

The URA is responsible for the enforcement and approval of the development plan for every land parcel in Singapore. It is also one of the main government agencies administering the land sale program. For every parcel of land offered for sale by open tender, the zoning and the density specifications are clearly dictated by the authority. In a fair and predictable environment, developers would put in their tender bids, given the knowledge of the use type and permitted density of the development for a particular site. Any change to the zoning and density of the site by the successful bidder would be subject to a hefty differential premium, a form of betterment levy on government land alienated on a 99-year lease. While these development control measures helped to ensure consistency and conformity in the physical development of Singapore, they are rigid and give developers limited room to exercise flexibility in their development plans. In areas that are undergoing rapid changes and development, it would not be feasible to a-priori fix the land use zoning and density of the site, unless the planning authority has all the information to "*pick the winners*" for the market.

In 1995, the URA introduced a category of land use known as "white" site. This is a significant step that relaxes the restriction on the use of land offered for sale by tender. Developers who successfully bid for these "white" sites are given the flexibility to determine the type of use, mix of use and also the quantum of different uses on the land without having to pay a differential premium so long as the permissible gross floor area is not exceeded. Under this new planning concept, the URA will only have minimum control on the development by defining the envelope of the building in the form of the plot ratio allowed, and also the range of uses permitted within the envelope. Developers will be allowed to determine the most optimal mix of use. The property market's response to the relaxation of the land use restriction has been positive, judging by the following comments:

"At the time planners pre-determine certain uses for sites, it's hard to know if this will meet the needs of the markets... Sometimes by the time a site is offered, the market may have changed.
The government's suggestion could correct a possible mismatch between the original intention of planners and the changing needs of the market."
Business Times (16 August 1995)

"The (white site) idea is innovative... because it allows developers to come up with their own concepts in an area.
Take Esplanade Mall (a white site), for example. It's a challenge to package a concept because there are plenty of offices, hotels and retail there already. It's more or less giving over the actual concept of the development to the developer."
Straits Times (10 May 1996)

The first two parcels offered for sale in 1995 under the new "white" site category were Land Parcel "A" at China Square and Land Parcel "C" at Middle Road. The two land parcels were offered on 99-year lease tenure and they are allowed for commercial or hotel or a mix of commercial and hotel uses. The 3,077.5 sq m China Square site, with a plot ratio of 13.86, was awarded to the Development Bank of Singapore (DBS) at the highest bid price of S$367.3 million (Singapore Dollars). The second white site at Middle Road, a commercial district at the fringe of the CBD, was won by a Malaysian listed company, IOI Properties, with a top bid price of S$52.2 million. It has a land area of about 2,600 sq m) and a plot ratio of 4.2.

Following the sale of these two sites, six other "white" sites were subsequently sold. The development details of the white sites and the successful bid prices of the sites are summarized in Table 1. Collectively, the eight white sites sold thus far have a total land area of 74,226 sq m and they are expected to yield more than 300,000 sq m of gross commercial, hotel and/or residential floor space when completed. In terms of total sale proceeds, a total of S$1.79 billion was collected from the sale of the eight white sites. The next white site in line for sale is a large site of 1.1 hectares located in the heart of the new downtown.[2] The Marina South site is expected to yield a total of 1.5 million sf of gross commercial / residential floor space to the market. In the future, more land parcels would be offered in the pipeline for sale under the "white site" concept. In fact, it was reported that 14% of the reclaimed land in the future new downtown, or about 50.3 hectares in land area, will be released as "white" sites over the next 10 to 15 years (Straits Times, 11 November 1997).

2. Under the new Downtown core, Strait View and Marina South DGP, the URA plans to expand the current CBD into the Central and Bayfront planning areas, which are on 139 hectares of reclaimed land. Together with the existing CBD, these areas when fully developed would form the future financial and business hub of Singapore.

Table 1: Details of White Sites Sold by Tender

Date of Award	Location	Allowable Development	Site Area (sqm)	Gross Plot Ratio	Maximum Permissible GFA (sqm)	Successful Tender Price ($)	Unit Price ($psm)
13/03/96	China Square LP(A) (Cross St/ Telok Ayer St.)	Commercial or Hotel or Commercial/ Residential	3,077.5	13.86	42,655	$367,310,736	$8,611.20
13/03/96	Middle Road/Prinsep St LP(C)	Commercial or Hotel or Commercial/ Residential	2,600.6	4.2	10,923	$52,222,222	$4,780.94
16/09/96	Raffles Link/ Esplanade Mall	Commercial or Commercial & Residential or Hotel Development with underground shopping mall	17,992.6 (LPA1: 8,782.8 LPA2: 9,209.8)	LPA1 3.5	36,739.8 (LPA1: 30,739.8 LPA2: 6,000)	$292,005,000	$19,753.02
21/10/96	Tekka Corner (Serangoon Road/Sungei Road)	Commercial or Hotel or Commercial & Service Apartment and Carpark Station	6,332	3.5	22,162	$84,000,000	$3,790.27
21/10/96	Cheang Jim Chwan Place LP(A) (Prinsep Street)	Commercial or Commercial and Residential or Institutional	1,385.30	Subject to gross floor area of conservation buildings	1,950#	$6,038,888	$4,359.26
06/11/96	China Square LP(F)	Commercial or Commercial & Residential or Hotel	13,981.70	3.5	48,550	$308,000,100	$6,343.98
04/06/97	China Square LP(G)	Commercial and/or Residential and/or Hotel and/or Medical Centre	13,554.10	3.0	40,300	$340,050,000	$8,437.97
21/02/00	Clarke Quay MRT Station	Commercial and/or hotel and/or residential or serviced apartment	15,302.3 (A1:11,321.5 A2:2,531.6 A3:724.6 A4:724.6)	5.6 (based on parcels A1 and A2)	77,577.36	$340,800,000	$4,393.03

* *LP denotes land parcel.* *sqm denotes square meter.* *$psm denotes $ per square meter.*

\# The combined supply of commercial space of China Square Parcel F and the Cheang Jim Chwan Place is approximated at 50,500 sm

Flexibility to switch use

The key element of the "white" site concept is the flexibility to switch the use or the quantum of the mix of use from time to time to optimize the rental value of the subject property. This should increase the development potential of the site, and hence enhance the market value of the site. The option to switch use is intuitively valuable, and the premium for this option should be fairly reflected in the value of the "white" site. The flexibility premium attached to the "white" site distinguishes it from a normal site and also enhances the residual land value of the "white" site.

The inter-temporal flexibility of switching the use and the quantum of the mix of use offers two economic benefits vis-à-vis the normal sites. First, there is a diversification benefit created when more than one use with a different mix quantum is allowed. If the rental streams generated by different uses are less than perfectly correlated, the fall in the rental income from one use type could be off-set by the increase in the rental revenue of another use type. The market risk will be higher in the case of a single use development when the market is heading towards an oversupply situation. The second benefit comes in the form of the dynamic flexibility in optimizing the mix of use for the development. When the demand for office space is weak, the developer could switch and convert some of the office space to retail use if the increase in the rental revenue justifies the exercise of the option. Further, the diversification and the dynamic flexibility benefits given by the switch use options can be enjoyed at any stage of development.

When the market is highly volatile and dynamic, the premium for the switch use option will be substantially increased. Therefore, it is economically significant to be able to determine the value associated with the flexibility to switch use and add this value to the base value estimated as if it is a normal site. It would also be interesting to provide a framework to evaluate the premium of the options, which, we believe, is not reflected in the traditional "highest and best use" definition. A two-stage sequential real options pricing model is proposed in this study to estimate the premiums associated with the switch use flexibility and it will be empirically applied to the valuation of the "white" sites sold by the URA.

MODEL SPECIFICATIONS AND BUILDING

Basic model

The options to switch use as discussed in the earlier section apply throughout the entire lease period of the subject site. The options are exercisable immediately after the legal transfer of the exclusive rights of use of the site to the developers. During the construction stage, the economic advantages of switching use that may involve changes to the original architectural design and plan are difficult to be determined and quantified. Therefore, we model the switch use options of a white site only for the post-completion period. For the pre-development stage, we incorporate in our model the optimal timing options of development for the white site. The switch use and the optimal development timing options could be modeled jointly and sequentially in a two-stage process. In other words, the option to switch the use of the completed floor space is only valuable when the option to develop has been optimally exercised. In this case, we assume a zero time lag in the construction process to simplify the structure of the proposed real options model, i.e. the construction takes place instantaneously upon exercising the option to develop.

The computation of the residual land value of the white sites with the embedded dynamic switch use and optimal timing options is carried out in a two-stage backward process. The value of the optimally scaled building that will be developed on the white site is first estimated with the dynamic switch use flexibility in mind. In the second stage, the optimal triggering rental ratio and the building value with the dynamic switch use options are included in the boundary conditions that determine the value of the "white" site together with optimal development timing options.

Stochastic processes for unit rentals

Assuming that the risk spanning condition holds in our real options model, appropriate stochastic processes have to be specified for the state variables. To model the switch use options in the optimally developed building, we assume that there are only two types of use (identified as *User 1* and *User 2*) with independent continuous time stochastic rental functions. Let x_1 and x_2 represent the gross rental revenue for leasing out one unit of building floor space to *User 1* and *User 2* respectively. The unit rental streams evolve in the following geometric Brownian motion process:

$$dx_i = \alpha_i x_i \, dt + \sigma_i x_i \, dz_i \qquad ; \qquad i = [1, 2] \tag{1}$$

where α_i and σ_i are the instantaneous drift and standard deviation of the cash flow stream, dz_i is an increment of the standard Wiener process with a normal distribution with a mean of zero and a standard deviation of 1, N(0,1). The random components of the rental variables are correlated by $[\rho_{12} = E\,(dz_1, dz_2)]$.

Options to switch use, $V(x_i)$ –stage one model building

The dynamic flexibility of repeatedly switching a particular type of use to an economically more favorable use type in response to the market condition provides ample room for a developer to optimize its cash inflow in a development and also to protect it from the downside risk of the market. In the shopping centre's tenant mix options, Grenadier (1995a) has demonstrated using a real options model how the value of the center could be enhanced through optimizing the tenant mix from time to time. Like the inter-store externality effects modeled by Brueckner (1993) in the space allocation exercise of a shopping center, the quantity effects in terms of floor space occupied by different tenants are also explicitly represented in Grenadier's (1995a) model to make the options to optimally mix the tenants more realistic. In another paper by Grenadier (1995b), which discusses the sticky tenancy and the adjustment costs in office leases, he adopts a fixed adjustment cost to proxy the owner's decision to change the status-quo of the occupancy condition in a building. Similarly, he adopts the same adjustment cost methodology in his tenant mix model as a triggering mechanism that determines whether the option to switch use or adjust the leasable space from one user to another would be exercised.

The switch use benefits offered by the "white" site are dependent on the inter-temporal fluctuation of rentals payable by tenants. The use of a relative rental as the state variable in Grenadier's model (1995a) restricts the full potential of the dynamic switch use options embedded in a project. Sing and Patel (2000) allow the rental streams of different users to be modeled independently by different correlated stochastic processes. This relaxation provides greater operational flexibility in the analysis of inter-temporal switch use options without having to impose a strict restriction of perfectly correlated rental volatilities for different tenants. For a "white" site development with only two users yielding two independent rental streams of x_1 and x_2, the switch use option model of Sing and Patel (2000) fits well in our attempt to estimate the dynamic flexibility embedded in "white" sites. Their model would be adopted in the first stage of our analysis to determine the optimal building value with options to switch use anytime within the lease period.

Let us start with a basic model having only two users. The rentals for the two tenants, x_1 and x_2, are assumed to follow two independent log-normal diffusion processes as in equation (1) with a correlation of ρ_{12} between the two random components, $\rho_{12} = E(dz_1, dz_2)$. The rental streams for *User 1* and *User 2* are dependent on the size of the floor space occupied, where the rental payoff $P(x_i,q_i)$, is no longer a monotonically increasing function of project scale. The rental payoff is a concave function of project scale, $P(x_i, q_i) = x_i.q_i^{\theta}$, where a price elasticity of scale, θ, that is less than unity rental revenue indicates a diminishing rate of growth of rental payoffs as the floor space occupied increases, i.e. $[dx_i/dq_i > 0; d^2x_i/dq_i^2 < 0]$. On the expenditure side, there is a lump sum fixed operating cost of (F_o) that covers the day-to-day running of the building regardless of the occupancy level of the building.[3] There is also a variable component of the operating cost that follows a convex relationship with the floor space occupied by the tenant (q_i). It is expressed as a Cobb-Douglas function cq^{γ}, where c is the unit variable cost and γ is the cost elasticity of density.[4] Based on the given payoff and cost functions, the building value without the dynamic switch use flexibility, $V(x_1,x_2)$, is simply equivalent to the accumulation of the net discounted revenue stream over the entire lease term, which is formally written as follows:

$$V(x_1,x_2)=[P(x_i,q_i)*3YP(t=99,n,e,i)]-\int_0^{\infty}[C(q_i)*\exp(-rt)dt]$$
$$=\frac{P(x_i,q_i)}{\delta}-\frac{C(q_i)}{r} \qquad (2)$$

where δ is the capitalization yield and r is the risk free rate. $P(x_i, q_i)$ and $C(q_i)$ are defined below:

$$P(x_i, q_i) = P_1+ P_2 = x_1(q_1)^{\theta 1} + x_2(1- q_1)^{\theta 2} - \beta_1 + q_1(\beta_2 - \beta_1) \qquad (3)$$

$$C(q_i) = c(q_1)^{\gamma 1}+ c(q_2)^{\gamma 2}+ F_o = c(q_1)^{\gamma 1}- c(1- q_1)^{\gamma 2} + F_o \qquad (4)$$

Two significant features of the model are the quantity and the interactive effects of space allocation on the rental revenue and operating costs. The measures of elasticity of scale, i.e. θ and γ, are used to represent the curvature of the rental (xq^{θ}) and operating cost (cq^{γ}) functions, where q denotes the quantity of floor space allocated, such that $\theta \leq 1$ implies a concave rental function, and $\gamma \geq 1$ reflects the convexity of the cost curve. The externality or interactive effect between two tenants is captured by the parameter β. The sign of β indicates the complementary ($\beta > 0$) or the competitive ($\beta < 0$) relationship

3. These expenditures include salaries for permanent site staffs, costs for periodic maintenance of mechanical and electrical equipment, service charges for basic facilities such as cleaning and security, promotional allocation (in the case of a shopping center project), insurance and tax liabilities.
4. The variation in the operating cost contribution for tenants with different sizes of leaseable space could also be adjusted by different γ factors subject to the conditions: $[(\gamma_1 \neq \gamma_2); (\gamma_1 \geq \gamma_2 | q_1 \geq q_2); (\gamma_1 \ \& \ \gamma_2 \geq 1)]$.

between the tenants, and the strength of the interaction is measured by the magnitude of β.[5] One assumption made in the "white" site valuation is that the proposed building will enjoy a full occupancy ($Q = q_1+q_2$) upon completion. With this imposition, the dynamic flexibility strategy will comprise two call options that allow density switching from one tenant to another. One option calls for an increase of *User 1*'s floor space by (Δq) to ($q_1 + \Delta q$), and another option reduces (q_1) by (Δq) which leads to a new tenant mix of [$Q = (q_1 - \Delta q) + (q_2 + \Delta q)$].

In an environment where rental stream evolves stochastically over time, the tenant mix is no longer a "one and for all" decision. The tenant mix that is optimal at time $t = 0$ will change from time to time, and the composition of the tenant needs to be dynamically adjusted to keep track with the volatility in the rental market. For example, when the office rental in the prime district declines, the existing tenant that pays an above market rental may opt to terminate his lease and relocate to another building with a lower rental. The landlord has a call option to convert or switch the space vacated to other uses such as retail, which may bring a higher rental. Compared to a static case where the use is fixed and not allowed to change after $t = 0$, a premium should, therefore, be added to the building value estimated in equation (2) to reflect the mix-use option features.

The combined values of the capitalized rental steam and the premium for the dynamic switch use flexibility are jointly estimated in a standard contingent claim valuation framework. The optimal value function for the completed building on a white site, $V = V(x_1, x_2, \sigma_1, \sigma_2)$, can be determined by solving the following partial differential equation (PDE)[6]:

$$\tfrac{1}{2} x_1^2 \sigma_1^2 V_{11} + \tfrac{1}{2} x_2^2 \sigma_2^2 V_{22} + \rho_{12}\sigma_1\sigma_2 V_{12} + (r - \delta_1)x_1 V_1 + (r - \delta_2)x_2 V_2 - rV + \pi(P,C) = 0 \qquad (5)$$

where $\pi(P,C)$ is the net interim cash inflows, [$\pi(P,C)=P(x_i, q_i) - C(q_i)$]. The above quadratic problem that involves more than one variable can be solved analytically by making the first-degree homogeneity assumption on the two rental variables, x_1 and x_2, i.e. let $R = x_1/x_2$ and $W(R) = V(x_1,x_2)/x_2$. The optimal value, $W(R^*)$, and the threshold value, R^*, can be jointly determined by numerically solving the PDE (5). R^* is the optimal ratio of rental x_1 over x_2, which triggers the option to switch Δq unit of leaseable floor space from *User 1* to *User 2*. Δq can take either a positive or a negative value. This option can be triggered anytime within the term of the lease by a pre-defined threshold rental ratio. The "exercise price," or more specifically the triggering mechanism, is determined by the change in value "before and after" the adjustment, $V_{q1}(P,C)$.

Let U define the upper limit of the threshold that triggers an increase of Δq unit of leaseable space for *User 1*, and R^u be the upper triggering ratio that encourages the owner to increase the allocation of Δq leaseable space to *User 1*. From equation (5), the

5. For example, $\beta = 1$ indicates a stronger interaction between the tenants than a $\beta = 0.5$, and a negative β implies a negative externality whereby the presence of one tenant reduces the desired space "utility" of other tenants in the building.
6. V_1 and V_2 represent the first order derivative terms of V with respects to x_1 and x_2. The same representation will apply throughout this paper unless indicated otherwise. For a general function, say F(x, y), we have
$F_1 = F_x = dF/dx$, and $F_{11} = F_{xx} = d^2F/dx_2$; $F_2 = F_y = dF/dy$, and $F_{22} = F_{yy} = d^2F/dy_2$.

building value with the switch-use option is analytically solved by the following equation system:

$$\left[A_1(q_1)(R'')^{\varphi_1} + A_2(q_1)(R'')^{\varphi_2} + \frac{V(q_1)}{x_2}\right] * (1+U) = A_1(Q_u)(R'')^{\varphi_1} + A_2(Q_u)(R'')^{\varphi_2} + \frac{V(Q_u)}{x_2} \quad (6a)$$

$$\left[\varphi_1 A_1(q_1)(R'')^{\varphi_1 - 1} + \varphi_2 A_2(q_1)(R'')^{\varphi_2 - 1} + \frac{(q_1)^{\theta_1}}{\delta}\right] * (1+U) = \varphi_1 A_1(Q_u)(R'')^{\varphi_1 - 1} + \varphi_2 A_2(Q_u)(R'')^{\varphi_2 - 1} + \frac{(Q_u)^{\theta_1}}{\delta} \quad (6b)$$

where:

$$\varphi_1 = \left[\frac{1}{2} - \frac{(\delta_1 - \delta_2)}{\sigma_R^2}\right] + \sqrt{\left[\frac{(\delta_2 - \delta_1)}{\sigma_R^2} - \frac{1}{2}\right]^2 + \frac{2\delta_2}{\sigma_R^2}} > 1 \quad (7a)$$

$$\varphi_2 = \left[\frac{1}{2} - \frac{(\delta_1 - \delta_2)}{\sigma_R^2}\right] - \sqrt{\left[\frac{(\delta_2 - \delta_1)}{\sigma_R^2} - \frac{1}{2}\right]^2 + \frac{2\delta_2}{\sigma_R^2}} < 0 \quad (7b)$$

Option to develop, $L(x_i)$ – stage two model building

Based on the same geometric Brownian motion assumptions given in equation (1) for the unit rental stream for *User 1* and *User 2*, we derive a real option model for the estimation of the premium associated with optimal development timing options for the white site. As the white site is sold as a vacant site, there is no interim cash flow generated during the construction period, i.e. $\pi = 0$. The value of the white site with options to wait to develop, $L(x_i)$, can be determined by analytically solving the following PDE:

$$0 = ½\, x_1^2 \sigma_1^2 L_{11} + ½\, x_2^2 \sigma_2^2 L_{22} + \rho_{12} x_1 x_2 \sigma_1 \sigma_2 L_{12} + (r-\delta_1) x_1 L_1 + (r-\delta_2) x_2 L_2 - rL \quad (8)$$

where ρ_{12} is the coefficient of correlation between the Wiener processes dz_1 and dz_2.

We can solve the PDE (8) by redefining the variables x_1, x_2 and the value function, $L(x_1,x_2)$ based on the first order homogeneity assumption, as $R = x_1/x_2$ and $M(R) = L(x_1,x_2)/x_2$. The new variables together with the first- and second-degree derivatives of M(R) are substituted into equation (8) to give us the ordinary differential equation (ODE) in the following form:

$$0 = ½(\sigma_1^2 + \sigma_2^2 - 2\rho_{12}\sigma_1\sigma_2) R^2 M_{RR} + (\delta_2 - \delta_1) R M_R - \delta_2 M \quad (9)$$

Equation (9) is a simple homogeneous second order differential equation that has the following general solution form:

$$M(R) = B_1 R^{\varphi 1} + B_2 R^{\varphi 2} \quad (10)$$

where φ_1 and φ_2 are given in equation 7(a) and 7(b).

M(0) = 0 is the absorbing barrier. The boundary conditions consist of the residual land value function $L(X_i)$, which is the difference between the capitalized value and the investment cost of I as in equation 11(a), and the first order differentiations of the residual land value function with respect to rental variables x_1 (equation 11b) and x_2 (equation 11c) respectively:

$$x_2 M(R) = L(x_i) = [(x_1 q_1^{\theta 1})/\delta_1 + (x_2 q_2^{\theta 2})/\delta_2] - I \quad (11a)$$

$$M_R(R) = L_1(x_i) = q_1^{\theta 1}/\delta_1 \quad (11b)$$

$$M(R) - RM_R(R) = L_2(x, \kappa) = 1/\delta_2 \quad (11c)$$

Based on the value matching condition (11a) and the first smooth pasting condition (11b), the solution for the ODE (9) can be derived at the optimal R^*, as follows:

$$M(R) = \begin{cases} B_1 R^{\varphi_1} & \text{for } R^* > R \\ \dfrac{Rq_1^{\theta 1}}{\delta_1} + \dfrac{q_2^{\theta 2}}{\delta_2} - I & \text{for } R^* \leq R \end{cases} \quad (12)$$

At R^*, the owner is indifferent between keeping the land vacant or to develop it. The closed form solution for the optimal land value, $L(x_1^*, x_2^*)$, as defined by equation (9) is derived in Sing (2001) as follows:

$$B_1 = \left(\frac{(q_2^{\theta 2}/\delta_2) - I}{1 - \varphi_1} \right)^{(1-\varphi_1)} * \left(\frac{q_1^{\theta 1}}{\delta_1 \, \varphi_1} \right)^{\varphi_1} \quad (13a)$$

$$R^* = \left(\frac{\varphi_1}{\varphi_1 - 1} \right) * \left(\frac{\delta_1}{q_1^{\theta 1}} \right) * \left(\frac{q_2^{\theta 2}}{\delta_2} - I \right) \quad (13b)$$

$$L(x_1^*, x_2^*) = x_2 * \left[\frac{(q_2^{\theta 2}/\delta_2) - I}{(1 - \varphi_1)} \right] \quad (14)$$

ESTIMATION OF OPTION PREMIUMS EMBEDDED IN "WHITE" SITES

Based on the theoretical real options models proposed in the earlier sections, we would proceed to apply the two-stage estimation procedure to determine the residual land values for the "white" sites embedded with dynamic switch use options and optimal development timing options. The values are then compared with those estimated by the deterministic discounted cash flow approach, which is known as the intrinsic land value. The difference in the values can be construed as the option premium for having the flexibility in switching uses and also the benefits to wait for the optimal time to initiate the development process. Five of the eight "white" sites given in Table 1 are selected for the empirical analysis as these have office as the predominant use.

Table 2: Common Input Parameters for Base Case Scenario

Input Parameter	Base Value	
User	1	2
Type of use	Retail	Office
Instantaneous drift (%)	$\alpha_1 = 5\%$	$\alpha_2 = 5\%$
Volatility (%)	$\sigma_1 = 25\%$	$\sigma_2 = 20\%$
Correlation for two Wiener processes, $E[dz_1, dz_2]$	$\rho_{12} = 0.0$	
Price elasticity of scale	$\theta_1 = 1.02$	$\theta_2 = 1.02$
Cost elasticity of scale	$\gamma_1 = 0.98$	$\gamma_2 = 0.98$
Coefficient for interaction	$\beta_1 = 1.1$	$\beta_2 = 1.1$
Risk free rate (%)	$r = 5\%$	
Lump sum fixed operating cost ($)	$F_o = \$0.0$	
Threshold limit for exercising option	$U = 0.2$	

* The URA Office and Shop Rental Indices for Central Area for the period 1989Q1 to 2001Q4 show an annualized rate of return of 3.62% and – 0.63% and an annualized standard deviation of 11.57% and 17.56% respectively. However, for the empirical analysis purposes, a 5% drift is deemed to be more representative for both office and retail rental, and the rental volatilities for office and shop rental are assumed to be 25% and 20% respectively. There are no empirical statistics for the elasticity of scales and coefficient of interaction parameters. The above assumptions are made to the best of the knowledge of the authors on the subject sites.

Input assumptions

For empirical estimation purposes, we only assume two types of use: retail (*User 1*) and office (*User 2*) for the "white" sites. Two sets of input parameters are separately assumed for the models. The first set of input parameters involves some general parameters that are common for all the "white" sites, which include the instantaneous drift rate and volatility of the rental streams for *User 1* and *User 2*, the correlation of the Wiener processes of the two rental streams, the price and cost elasticity of scale, the coefficient of interaction, risk free rate, lump-sum fixed operating costs and threshold for the switch use options. These parameters form the base case scenario of our analysis and are summarized in Table 2. The second set of parameter inputs is case-specific and case-dependent. The parameters are obtained based on the inspection and surveys carried out on the five selected sites. However, the information on the existing and proposed uses and their development status, together with our assumptions with respect to the allocation of usable space between the uses, unit rental, unit variable operating costs, capitalization yields and costs of development is included in Table 3. The development costs comprise both the land cost and the construction costs.

Table 3: Case Specific Input Assumptions in Base Case Scenario for the Subject Sites

Subject White Site	China Square LP "A" (Cross St/ Telok Ayer St.)	IOI Plaza (Middle Road/Prinsep St LP "C")	Tekka Corner (Serangoon Road/Sungei Road)	China Square LP "F"	China Square LP"G"
Development Status	The PWC Building –a 28-storey with a 5-storey carpark podium completed in 1999	IOI Plaza – a 16-storey commercial building completed in Nov. 1998.	Vacant and awaiting for development	A 15-storey commercial building is under construction and 32 units of conservation shop-house are being restored	A 16-storey commercial building is under construction and 63 units of conservation shop-house are being restored
Hypothetical allocation of space between two users	Office: 90% Retail: 10%	Office: 80% Retail: 20%	Office: 70% Retail: 30%	Office: 70% Retail: 30%	Office: 60% Retail: 40%
Option scenario:					
Total Built-up Area (sq m), Q	42,639	10,919	22,154	48,532	40,285
Floor space allocation for User 1 (retail) (sq m), q_1	4,264	2,184	6,646	14,560	16,114
Floor space allocation for User 2 (office) (sq m), q_2	38,375	8,735	15,508	33,972	24,171
		A) Switch Use Option			
Switching space between users	$Q = (q_1+\Delta q) + (q2 - \Delta q)$	$Q = (q_1+\Delta q) + (q2 - \Delta q)$	$Q = (q_1+\Delta q) + (q2 - \Delta q)$	$Q = (q_1+\Delta q) + (q2 - \Delta q)$	$Q = (q_1+\Delta q) + (q2 - \Delta q)$
Qty of floor space being switched (sq m), Δq	2,132	546	1,108	2,427	2,014
Unit monthly gross rental for user 1 ($psm), x_1	$75.35	$64.60	$59.20	$74.30	$74.30
Unit monthly gross rental for user 2 ($psm), x_2	$62.40	$45.20	$43.10	$61.40	$62.40
Variable operating costs ($psm), c	$8.60	$6.50	$6.50	$8.60	$8.60
Capitalization rate for user 1 (Gross), δ_1	8.75%	8.75%	9.00%	8.75%	8.75%
Capitalization rate for user 2 (Gross), δ_2	5.75%	6.00%	6.00%	5.70%	5.75%
		B) Option to Develop			
Unit land cost ($psm)	$8,614	$4,782	$3,792	$6,346	$8,441
Unit construction cost ($psm)	$3,767	$2,691	$2,691	$3,767	$3,767

** LP denotes land parcel. sqm denotes square meter. $psm denotes $ per square meter.*

Analysis of results

Based on the above input assumptions, the optimal building values given the option triggering relative rental R^* are estimated in a full occupancy condition. The optimal rental ratio, R^*, is then used to estimate the residual land value embedded with the optimal development timing options. The results are summarized in Table 4.

In the region where $R > R^*$, the proposed mix of *User 1* and *User 2* is optimal for the respective "white" sites. The net present value for the subject "white" site is determined by deducting the costs of land and construction from the value of the completed property. This value reflects not only the expected return of the developer for undertaking the risk of development; it also contains the value of the flexibility to switch use and also the option to choose the optimal time of development. By comparing the estimated optimal value for the subject "white" site with the intrinsic value, which is the deterministic discounted value assuming that rental volatilities for office and retail users are zero, the values of the optimal development timing options and the switch use options can be estimated by taking the difference between the value obtained by the real options model and the intrinsic value. The premiums for both the switch-use and optimal development timing options are also computed as a percentage of the non-stochastic land values. The results are shown in Table 4.

Table 4: Results of Option Premium Estimations

Subject Development	China Square (A)	IOI Plaza /Middle Road	Tekka Corner/ Serangoon Rd	China Square (F)	China Square (G)
a) Option to Switch Use					
Optimal building value with switch use option, $V(x_1, x_2)$	$ 742,490,475	$131,275,636	$252,071,428	$800,321,449	$650,585,973
Building value in a deterministic scenario, $V(\sigma_i=0)$	$ 702,557,428	$123,449,464	$236,542,692	$756,451,941	$614,210,831
Option premium:					
Optimal rental ratio, R^*	1.21	1.43	1.38	1.21	1.19
In value term ($)	$ 39,933,047	$ 7,826,172	$ 15,528,735	$ 43,869,508	$ 36,375,142
As a percentage of deterministic value (%)	5.68%	6.34%	6.56%	5.80%	5.92%
b) Option to Wait to develop					
Unit construction cost	$350	$250	$250	$350	$350
Total Construction cost	$ 160,638,730	$ 29,382,875	$ 59,615,775	$182,839,300	$151,769,800
Actual Tender price	$ 367,310,736	$ 52,222,222	$ 84,000,000	$308,000,100	$340,050,000
Residual Land Value:					
With option premium, $L(x_1,x_2)$	$ 581,851,745	$101,892,761	$192,455,653	$617,482,149	$498,816,173
Deterministic case $L(\sigma_i=0)$	$ 541,543,234	$ 93,581,601	$175,836,563	$572,493,631	$461,276,345
NPV (Real option model)	$ 214,541,009	$ 49,670,539	$108,455,653	$309,482,049	$158,766,173
NPV (Deterministic model)	$ 174,232,498	$ 41,359,379	$ 91,836,563	$264,493,531	$121,226,345
Option premium:					
In value term ($)	$ 40,308,511	$ 8,311,161	$ 16,619,089	$ 44,988,518	$ 37,539,829
As a percentage of deterministic value (%)	23.13%	20.09%	18.10%	17.01%	30.97%

The results show that the switch use option premium ranges from 5.68% for the China Square parcel "A" site to 6.56% for the Tekka Corner site. In absolute value term, the Middle Road site has the lowest switch use option premium of S$7.83 million, whereas the China Square parcel "F" site is found to have the highest premium of S$43.87 million. The results also show that the option premium increases when the switch use option premium is combined with the optimal development timing benefits. The total option premium ranges from 17.01% for the China Square parcel "F" site to 30.97% for the China Square parcel "G" site. In absolute term, the option values range from S$8.31 million (Middle Road site) to S$44.99 million (China Square parcel "F").

Sensitivity analysis

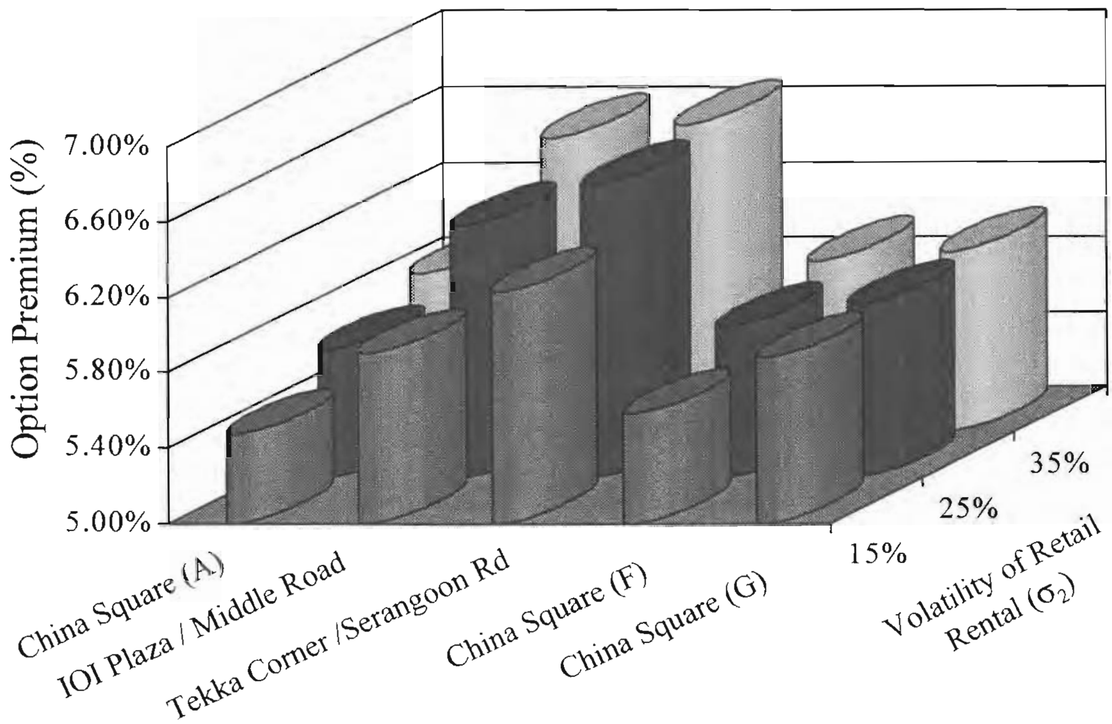
The results given in Table 4 are based on the parameters given in Table 2. This section further examines the effects of the changes in the rental volatility on the option premium estimates. The rental volatility, x_1 and x_2, of *User 1* and *User 2* respectively, is varied by plus and minus 10%, i.e., 15% to 35% for x_1 and 10% to 30% for x_2. The results of the sensitivity analysis are summarized in Table 5. The switch use option premium now ranges from 4.85% (lowest) to 6.67% (highest), which is slightly wider than the fixed volatility range analyzed earlier. When the switch use option premium is combined with the optimal development timing benefits, the total option premium ranges from 15.65% to 31.4%, which is again wider than the premium under fixed volatility.

Table 5: Effects of Changes in Rental Volatility on the Option Premiums

			Proposed / Completed Developments for the White Sites				
Office Rental Volatility (σ_2)	Retail Rental Volatility (σ_1)	Option Type*	China Square (A)	IOI Plaza /Middle Road	Tekka Corner	China Square (F)	China Square (G)
10%	15%	I	5.13%	4.85%	6.07%	5.79%	5.78%
		II	20.91%	15.65%	17.86%	16.99%	30.36%
	25%	I	5.72%	6.22%	6.15%	5.81%	5.82%
		II	23.26%	19.73%	19.61%	17.04%	30.43%
	35%	I	5.89%	6.62%	6.35%	5.88%	6.00%
		II	23.97%	20.92%	20.11%	17.25%	31.38%
20%	15%	I	5.47%	5.90%	6.23%	5.58%	5.88%
		II	23.59%	20.52%	20.01%	16.75%	29.56%
	25%	I	5.68%	6.34%	6.56%	5.80%	5.92%
		II	23.13%	20.09%	18.10%	17.01%	30.97%
	35%	I	5.85%	6.57%	6.64%	5.91%	5.96%
		II	23.82%	20.79%	18.28%	17.32%	31.16%
30%	15%	I	5.73%	6.43%	6.59%	5.82%	5.94%
		II	23.30%	20.35%	18.16%	17.07%	31.03%
	25%	I	5.83%	6.56%	6.63%	5.89%	5.96%
		II	23.74%	20.77%	18.27%	17.28%	31.15%
	35%	I	5.96%	6.67%	6.66%	5.96%	6.01%
		II	24.23%	21.09%	18.33%	17.47%	31.40%

* *Option type I refers to the switch use option*
Option type II refers to the optimal development timing option

Figure 1: Sensitivity Analysis for the Switch Use Option
(Office rental volatility, σ_2 = 20%)



The variation in the switch use option premiums for the scenario with σ_1 = [15%, 25%, 35%] and σ_2=20% is graphically shown in Figure 1. The results are consistent with the real option theory. The option premium increases with the corresponding increase in the volatility for all the five subject sites. It implies that the options to wait and also to switch use are more valuable in time of greater uncertainty.

CONCLUSION

The new "white" site land use zoning is aimed at providing greater flexibility for developers to maximize the potential of the sites they bought via the government's sale of sites program. Endowed with the flexibility to switch use, developers can respond to the market demand and supply conditions more effectively by instantly adjusting and optimizing the space between different uses without having to pay a hefty differential premium. Eight "white" sites have so far been offered for sale by tender. However, the tender results seem to indicate that market analysts have not fully evaluated the economic advantages provided by the switch use flexibility. This could be due to the methodological limitations inherent in the residual land valuation and/or the discounted cash flow techniques currently used by the industry.[7] To overcome the technical

7. Another reason suggested by one of the anonymous referees is that the analysts or developers may perhaps hold a more conservative view on the rental volatilities, which are the critical factors in determining the switch-use and optimal development timing option premiums.

limitations in the traditional valuation techniques, real option models encompassing a switch use option and an optimal development timing option are derived in a two-stage process in this study.

The proposed real option models are then applied to five white sites where the flexibility in switching the space from a dominant use (office) to an alternative use (retail). The results show that the China Square parcel "A" site has the lowest switch use option premium of 5.68%, whereas the Tekka Corner site has the highest premium of 6.56%. The switch use option is computed to be worth from S$7.83 million (Middle Road site) to S$43.87 million (China Square parcel "F"). The option value increases to S$8.31 million for the Middle Road site and S$44.99 million for the China Square parcel "F" site when the option to decide on the most optimal time to develop is also included.

The sensitivity analysis shows that by varying the rental volatility of x_1 and x_2 by both plus and minus 10%, the switch use option premiums change from the lowest of 4.85% to the highest of 6.67%, whereas the premiums for the optimal development timing options swing by a wider range from 15.65% to 31.40%.

Our proposed models estimate the positive premiums for switching the allocation of space from a major use to a minor use with a smaller proportion of the total space but commands a higher unit rent. The converse could also happen where the developer converts part of the floor space from a user paying a higher rent for another use with a lower rent. This scenario is common in a glut market, where it is difficult to secure tenancy for a particular type of use. Therefore, it would be a disincentive for the developer to continue to keep the space vacant with the hope that he would secure a tenant at the market rate. If the rental revenue of the building drops sufficiently to trigger a negative switch of use from one to another at a lower rent, the options to switch in this type of scenario help to minimize the economic losses and also protect the downside of the investment. The downward switching of use from retail (which is the dominant use in the other three "white" sites not analyzed in this study) to office, which commands a lower market rent, i.e. $R<1$, is also a type of valuable option, especially in volatile market.

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